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QUASI-1D DEFORMATION MODELLING OF A TSF UNDER UNDRAINED LOADING

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Abstract. The assessment of tailings static liquefaction has become a key topic in recent years. The stability of upstream-raised tailings dams relies on the strength of tailings, which are loose and normally consolidated materials that may exhibit strain-softening during undrained loading. Limit equilibrium analyses do not consider the work input required to drive the softening process that leads to progressive failure, and therefore require that only fully softened strength be considered in the analyses. On the other hand, FEM analysis can capture the complete stress path of the softening behavior, but may lead to geometrically complex and computationally expensive models. This paper describes a simplified procedure to analyze the stability of tailings storage facilities employing a quasi-1D soil column using the constitutive model NorSand. A fully automated script was developed to produce meshes of various heights and characteristics, to reproduce the entire deposition sequence, and to perform push over analyses. The proposed strategy reproduces the main features of tailings behavior along the relevant stress path, estimates the brittleness of tailings at various depths, and produce an assessment of the risk of static liquefaction. It is shown that this procedure is useful to verify the stability of the TSF at screening level, and to provide guidance in the optimization of the dam design.

1 INTRODUCTION

Tailings Storage Facilities (TSF) are hydraulically deposited, non-compacted reservoirs of tailings, a rock flour which is a by-product of mineral extraction. Upstream-raised TSFs are built using the tailings themselves as construction materials, thus relying in the strength of a loose granular material which may liquefy in undrained loading. International guidelines recommend that flow (static) liquefaction of TSFs be analyzed taking into account the brittle/contractive nature of saturated tailings. Thus, quantifying the material contractiveness is of the utmost importance in analysis and design of TSFs.

In TSF design, FEM analyses may lead to geometrically complex and computationally expensive models. For conventional slopes, an infinite slope analysis provides much of the insight required to guide design. Dealing with brittle materials, conventional limit equilibrium methods cannot be employed reliably, and therefore the infinite slope analysis must be replaced by its numerical counterpart, where just a slice of the TSF is modelled. In this way, the problem can be analyzed as a quasi-1D column representative of the full TSF for screening purposes. As the geometry is simplified and the computational cost is greatly decreased, the entire model can be created and run using Python within the PLAXIS 2D® scripting environment, allowing for the analysis of multiple scenarios and geometries, and providing a tool to incorporate the uncertainty of the material parameters into the design process.

In this paper, the staged construction of the TSF, $p' - e$ evolution, self-weight stability, and its pushover stability isanalyzed for a quasi-1D column. The NorSand constitutive model (Jefferies, 1993) implemented in PLAXIS 2D® is used. The importance of K_0 and its implication for the calibration of NorSand is discussed. Finally, the results for push-over analysis are explored and conclusions are discussed.

2 NORSAND MODEL

The volume change behaviour of granular materials depends on the void ratio e and mean effective stress p', and is captured by constitutive models that evolve e as a function of p'. The state parameter ψ (Been and Jefferies (1985)) is widely used to quantify the tendency of the material to contract or dilate in shear.

The NorSand (NS) constitutive model is a critical state soil model very well-known in the industry and used for static liquefaction modelling. In NS, the state of a soil is determined by two variables: the state parameter ψ and the image pressure p_{im}. The first one is defined as $\psi = e - e_c$, being e the current void ratio and e_c the critical state void radio at the current mean effective pressure. The second one is the mean pressure at which the volumetric plastic strain rate is zero ($\dot{\epsilon}_v^p = 0$, defined as the image condition i). NorSand assumes associative plasticity (see Eq. (1)) (Vermeer and de Borst, 1984).

$$\text{Flow rule: } \dot{\boldsymbol{\epsilon}}^p = \lambda \frac{\partial f}{\partial \boldsymbol{\sigma}} \tag{1}$$

Figure 1 shows the yield function of NorSand. p_{mx} is defined as the mean stress when the maximum dilatancy occurs, that is to say, where the dilatancy term D^p is minimum.

The relationship between strength and dilatancy is computed following Nova and Wood (1982)

$$\eta_{max} = M_{cv} - (1 - N) D^p_{min} \,, \tag{2}$$

where $\eta = \dfrac{q}{p'}$ is the stress ratio and N is a density-independent material property relating to volumetric work.

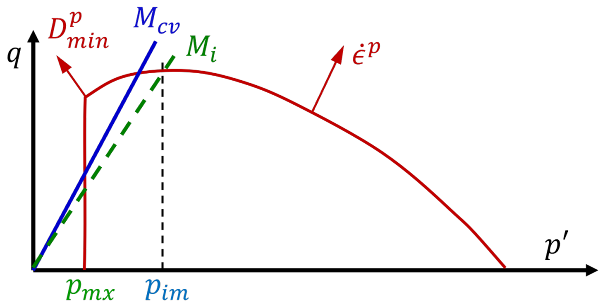

Figure 1: NorSand yield function

Considering the linear relationship between D^p and ψ

$$X_i = \frac{D^p_{min}}{\psi_i} , \tag{3}$$

where the subscript i denotes the image state. Then the zero-dilatancy 'frictional dissipation' stress ratio results in

$$M_i = M_{cv} - NX_i\psi_i , \tag{4}$$

where M_{cv} corresponds to critical state friction ratio in triaxial conditions.

The NorSand dilatancy term is computed using

$$D^p = M_i - \eta , \tag{5}$$

while the yield function is defined by

$$p' > p'_{mx} \rightarrow \frac{\eta}{M_i} + \ln\left(\frac{p'}{p'_{im}}\right) - 1 = 0 . \tag{6}$$

The critical state locus (CSL) is defined by

$$e_c = \Gamma - \lambda_e \cdot ln(p') , \tag{7}$$

where Γ is the void ratio corresponding to a mean pressure equal to $1kPa$, and λ_e is the slope of the critical state line (in $e - ln(p)$ space) and the hardening function is

$$\dot{p_{im}}' = \left[H \frac{p'}{p'_{im}} \frac{M_i}{M_{tc}} (p'_{mx} - p'_{im}) - S_{soft}\right] \dot{\epsilon}^p_q , \tag{8}$$

where

$$p'_{mx} = p' \cdot \exp\left(-\frac{\chi_{tc}}{M_{tc}}\psi\right) , \tag{9}$$

$$H = H_o - H_\psi \psi , \tag{10}$$

where H_o and H_ψ are dimensionless and determined by optimizing a set of drained triaxial tests which include loose and dense states. Additionally, the softening term is

$$D^p \leq 0 \rightarrow S_{soft} = S = 0 , \tag{11}$$

$$D^p > 0 \rightarrow S_{soft} = S \cdot \omega \left(\frac{\eta}{M_i}\right)\left(\frac{K}{p}\right) D^p p_{im} , \tag{12}$$

where

$$\omega = 1 - \lambda_e \frac{X_{tc}}{M_{tc}} \quad \text{and} \quad S = 1 \,, \tag{13}$$

while the bulk modulus is

$$K = \frac{2}{3}\left(\frac{1+\nu}{1-2\nu}\right) G \,, \tag{14}$$

being ν the Poisson modulus and the elastic shear modulus is

$$G = G_{ref}\left(\frac{p'}{p_{ref}}\right)^{n_G} \,, \tag{15}$$

where n_G is an exponent for a power-law trend and p_{ref} is the reference pressure, commonly equal to $100kPa$ as a widespread convention.

Additionally, NorSand is able to represent over-consolidated states with the parameter R, which is the over-consolidation ratio, and pushes the yield surface away from the current stress state.

NS has the ability to represent both dilative and contractive behaviour. As it uses only one yield function, when the model is calibrated to capture shear behaviour, the compression stress path is not correctly reproduced. In order to model both behaviours it is necessary to calibrate two sets of parameters, one for the staged construction of the TSF, where compression stress paths prevail, and another for the stability analysis, where shear stress paths prevail. This is possible by simply calibrating different M_{cv} parameters for each stage.

3 MATERIAL PARAMETERS

Table 1 shows the base material parameters used in this work. The same parameters from the examples of the PLAXIS 2D® manual were used, only modifying the rigidity and the CSL slope to match values closer to tailings materials.

G_{ref}	p_{ref}	n_G	ν	Γ	λ_e	M_{tc}	N	χ_{tc}	H_0	H_ψ	R	S
20000	100	0.5	0.2	1	0.06	1.3	0.35	4	300	0	1	1

Table 1: Material properties

3.1 Undrained triaxial compression

In Figure 2 the behaviour for undrained triaxial compression is shown. When $\psi < 0$ the material is dilative, and hence strain hardening behaviour is observed. On the other hand, $\psi > 0$ produces a contractive behaviour and strain softening.

3.2 Influence of K_0

$K_0 = \dfrac{\sigma_3}{\sigma_1}$ is a widely used indicator of stress obliquity in oedometric compression, determining the starting obliquity of the deviatoric loading stage and influencing the stress evolution during undrained softening, as shown in Figure 3. The importance of K_0 for static liquefaction is discussed in Fourie and Tshabalala (2005), where the effect of K_0 on the instability line is shown.

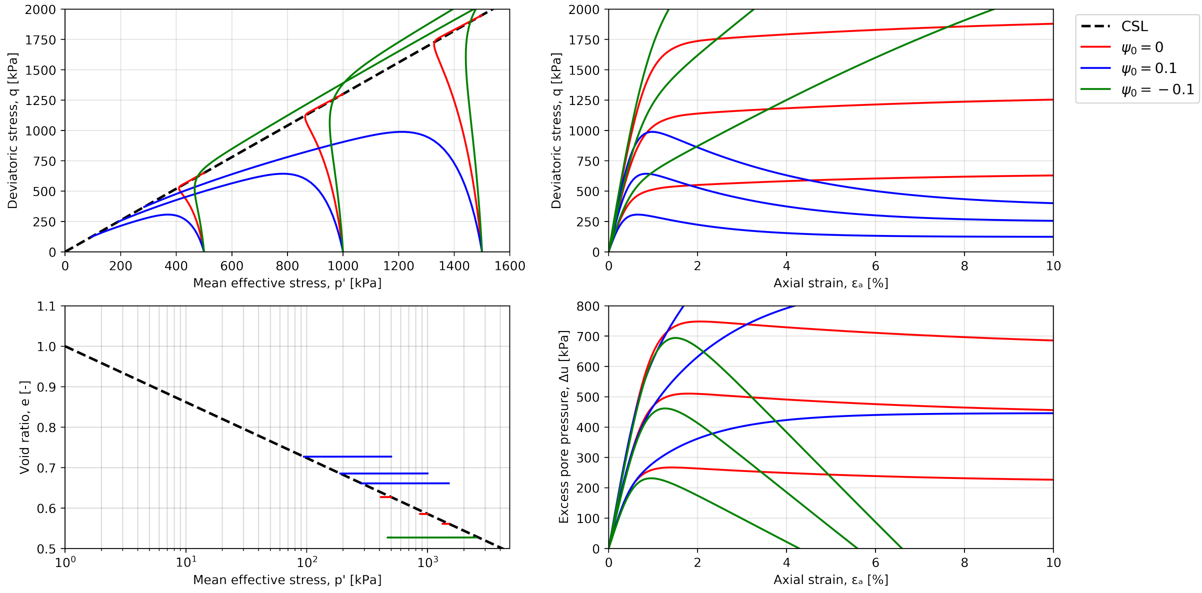

Figure 2: Effect of ψ on undrained triaxial compression simulation

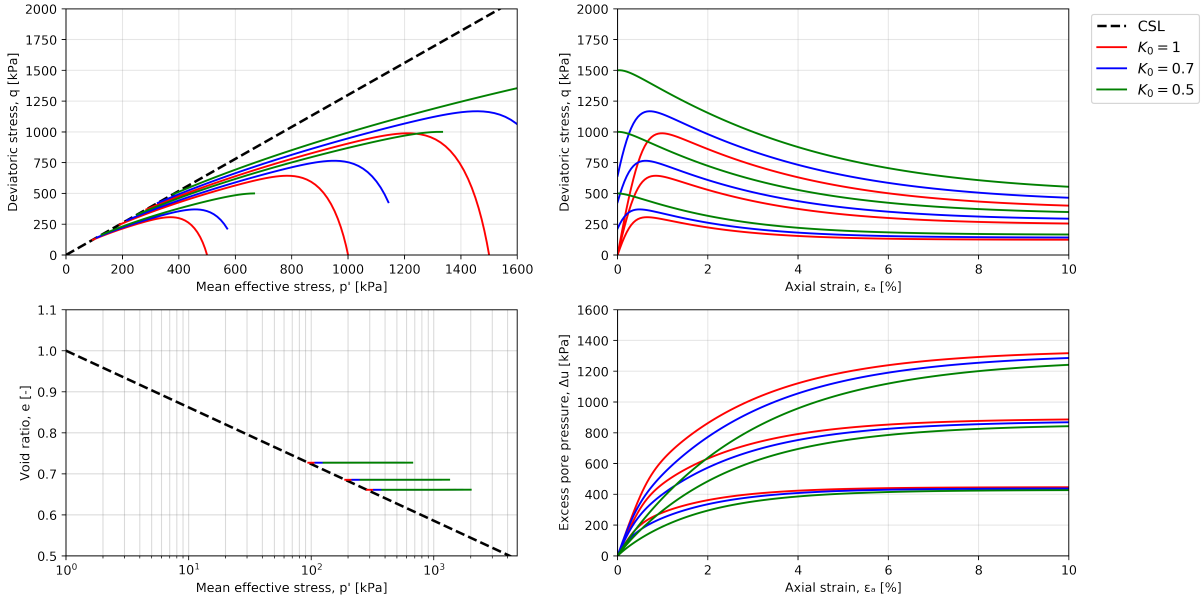

Figure 3: Effect of K_0 on undrained triaxial compression ($\psi_0 = 0.1$)

3.3 Influence of M_{cv}

A set of parameters calibrated for undrained triaxial compression results in poor adjustment to proportional compression paths and unrealistic values of K_0. To solve this, a different value M_{cv} must be be chosen for modelling staged construction. The effect of M_{cv} on K_0 is shown in Figure 4. A value of $M_{cv} = 1.7$ was chosen to yield $K_0 \approx 0.65$. All other material parameters may be kept unchanged.

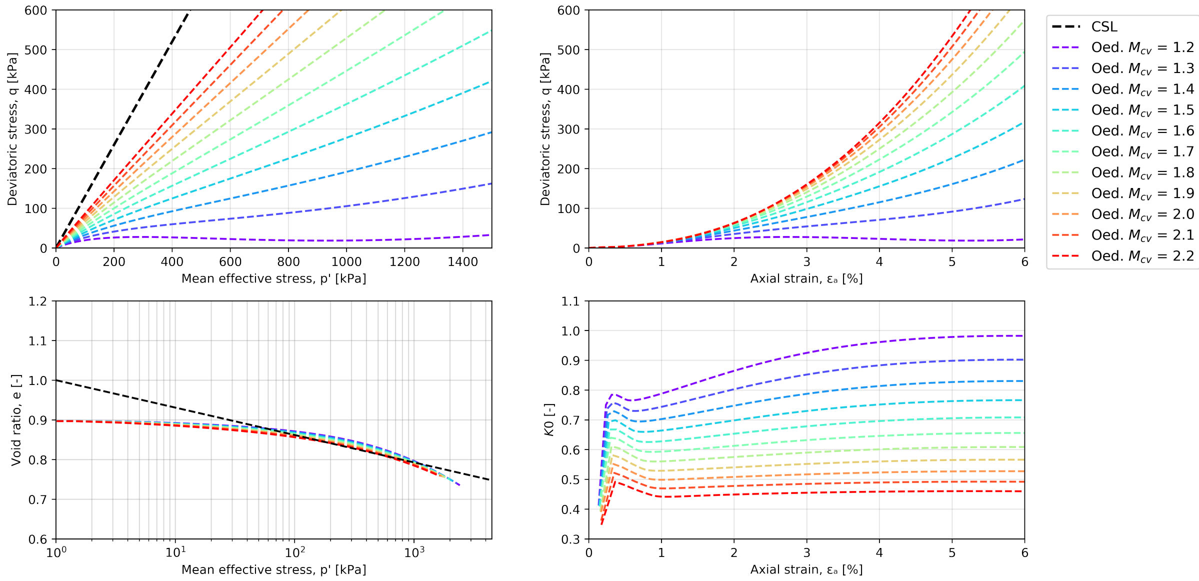

Figure 4: Effect of M_cv on triaxial tests with isotropic consolidation

4 NUMERICAL ASSESSMENT OF A TSF

4.1 Model description

The full staged construction until its final configuration of a slice representative of a large TSF was simulated, and a pushover undrained analysis was carried out. The detailed sequence, as shown in Figure 5, was:

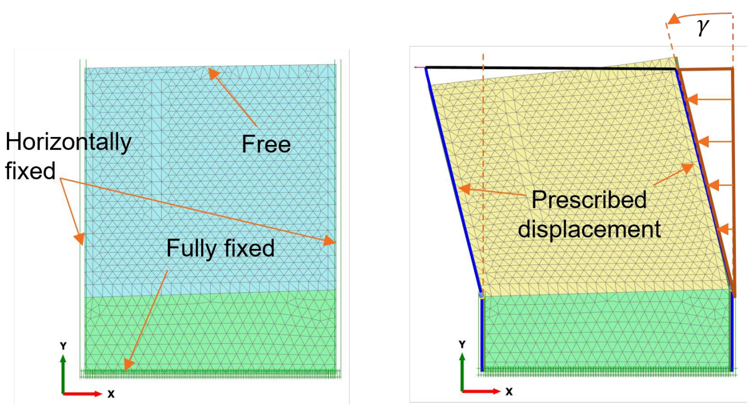

Figure 5: Typical model (size and number of layer vary along realizations)

- Construction: tailings stages raises were carried out assuming drained behaviour. Normally fixed lateral borders were imposed as boundary conditions. A phreatic level was applied to consider a saturated region. The material used in this steps was calibrated to capture compression deformation-paths.

- Stability analysis: tailings were forced to deform under undrained conditions. A linear lateral displacement was imposed on the borders to generate a uniform distortion γ at the boundaries and the horizontal component of the reaction forces (ΣF_H) were measured. A typical ΣF_x-γ plot is shown in Figure 6, where three regions can be identified. Initially, the column tries to find a self-equilibrium, and tends to move itself to the left. As the lateral displacements are prescribed, the reaction points to the right side, defining region 1. When the self-equilibrium is reached, reaction forces vanish ($\Sigma F_x = 0$), defining the boundary between region 1 and region 2. In region 2, an additional force is required to grow the prescribed displacement to the left side. This region ends at the point where the column is no longer available to resist the displacement due to strain softening, and hence is no longer stable ($\Sigma F_x = 0$). After this point, the column has to be sustained by left-oriented forces, defining the region 3. Hence, as a summary, the distortion when $\Sigma F_H = 0$ for the first time, γ_0, corresponds to the position at self-equilibrium, the second time, at γ_f, corresponds to the onset of instability. For the analyses shown in this paper, a maximum distortion of 5% was imposed and the full ΣF_x-γ path was captured.

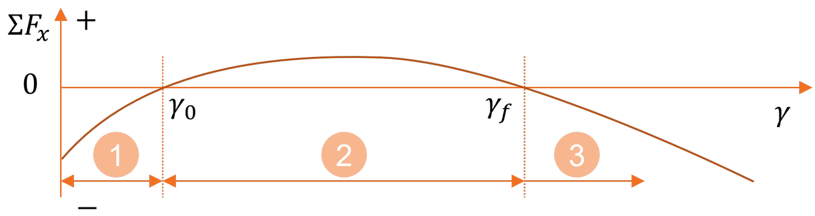

Figure 6: Interpretation of pushover analysis

Note: In all stages for the topmost layer a Hardening Soil model with Small stiffness constitutive model was used due to NorSands instability at $p' \approx 0$.

4.2 Model strategy

A fully automated script was developed to produce meshes of various geometries, changing the width B, foundation height H_f, number of layers N_t, thickness t_t, tailings inclination β, and a position of water table H_w. In the examples shown here the parameters chosen were: $B = 10m$, $H_f = 30m$, $N_t = 6$ and 12, $t_t = 5.0m$ and $1.5m$, $\beta = 3\%$, and $H_w = 9m$ and $18m$ (corresponding to the 30% and 60% of the height of the tailings dam respectively). Two different values for the thickness t_t were used to analyse the effect of the stages and layers employed to model the construction procedure. Some model geometries and meshes are shown in Figure 7.

4.3 Results

A summary of the results are shown in Table 2, where e_0 is the initial void ratio, ψ_0 is the initial state parameter, γ_0 is the distortion at static equilibrium, and γ_f is the distortion at the

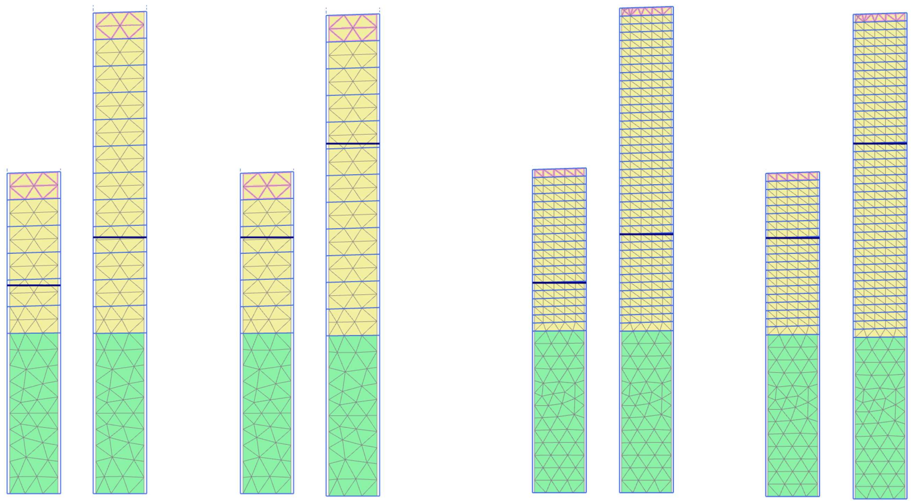

Figure 7: Analyzed Meshes. (a) Coarse layers, (b) Fine layers

onset of failure. Full ΣF_x-γ curves for $\psi_0 = -0.1$ can be seen in Figure 8 and for $\psi_0 = 0.1$ in Figure 9. It is shown that a higher ψ_0 leads to worse stability indicators and higher brittleness in undrained shear.

			$H_w = 9m$		$H_w = 18m$					$H_w = 9m$		$H_w = 18m$	
$e_0[-]$	$\psi_0[-]$	$H[m]$	$\gamma_0[\%]$	$\gamma_f[\%]$	$\gamma_0[\%]$	$\gamma_f[\%]$	$e_0[-]$	$\psi_0[-]$	$H[m]$	$\gamma_0[\%]$	$\gamma_f[\%]$	$\gamma_0[\%]$	$\gamma_f[\%]$
0.7	-0.3	30	0.03	-	0.04	-	0.7	-0.3	30	0.04	-	0.06	-
		60	0.07	-	0.10	-			60	0.10	-	0.11	-
0.9	-0.1	30	0.02	-	0.05	-	0.9	-0.1	30	0.07	1.07	0.08	2.54
		60	0.10	-	0.11	-			60	0.17	1.22	0.18	2.04
1.1	0.1	30	0.14	1.10	0.20	2.38	1.1	0.1	30	0.06	1.07	0.14	1.00
		60	0.22	3.95	0.24	3.91			60	0.22	1.49	0.24	1.08

Table 2: Pushover results. (a) Coarse layers, (b) Fine layers

Void ratio and state parameter at the end of deposition are shown in Figure 10 for $\psi_0 = -0.1$ and in Figure 11 for $\psi_0 = 0.1$. It is shown that the change of void ratio with depth was captured. However, when the height of each layer is not small enough, a non-smooth distribution of ψ_0 can be observed. This highlights the importance of simulating the staged construction in many thin phases. As it can be seen in Table 2, the size of the layer is significantly relevant to determine the stability of the columns. Using the current automate modelling scheme this means only changing the inputs, so no extra effort from the modeler is needed, just more calculation time.

A few conclusions can be drawn from the results:

- The change in the void ratio and state parameter due to the 1D primary compression was captured by the model.
- The effect of the initial void ratio was correctly captured. As the initial void ratio increases so does the likelihood of collapse, and a smaller strain is need for this to occur.
- The importance of modelling the deposition using small layers to accurately calculate the distribution of the void ratio and state parameter was shown.
- In all the cases analyzed, the tailings column was stable for self-weight.

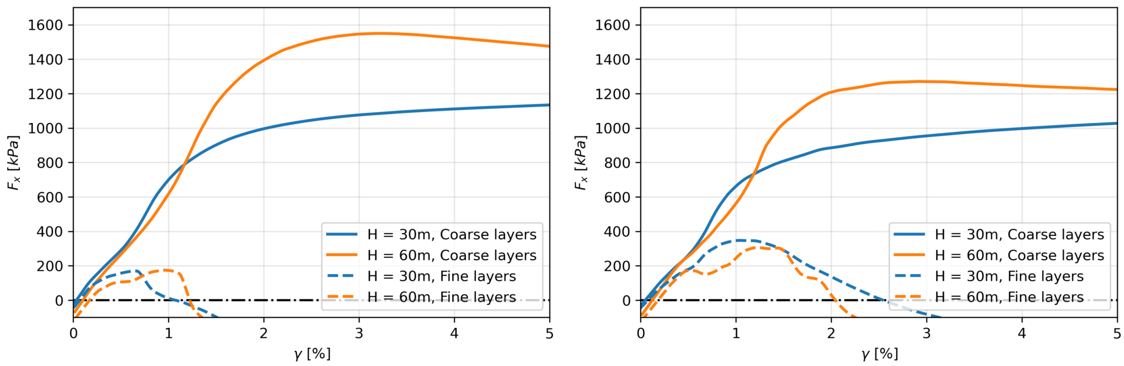

Figure 8: Pushover $\psi_0 = -0.1$. Water table at 30% (a) and 60% (b)

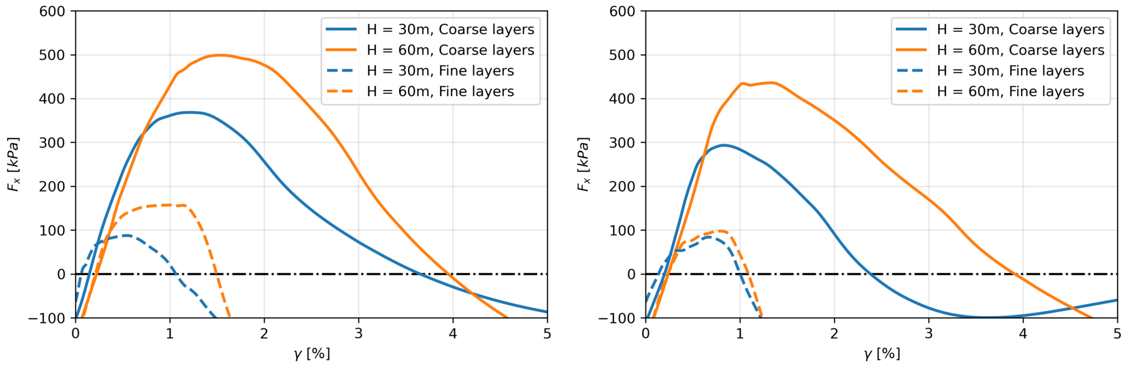

Figure 9: Pushover $\psi_0 = 0.1$. Water table at 30% (a) and 60% (b)

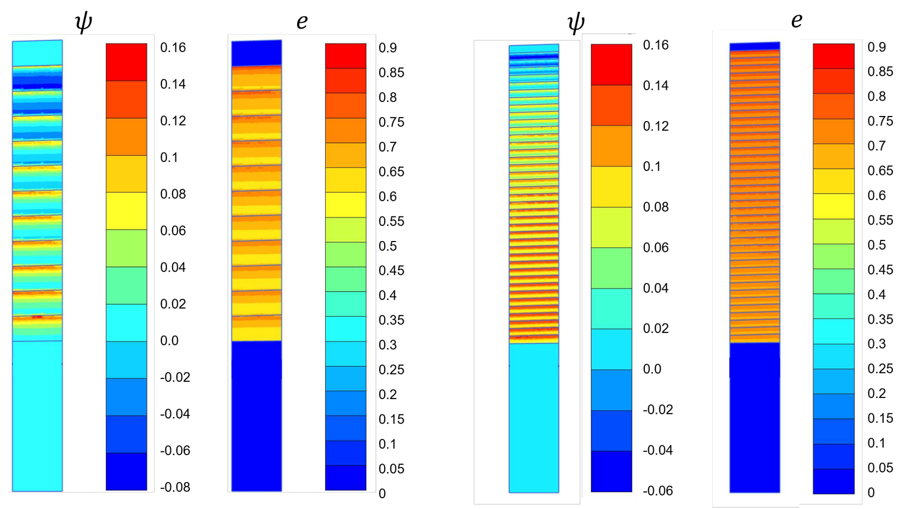

Figure 10: Void ratios and state parameters at the end of deposition ($\psi_0 = -0.1$). (a) Coarse layers, (b) Fine layers

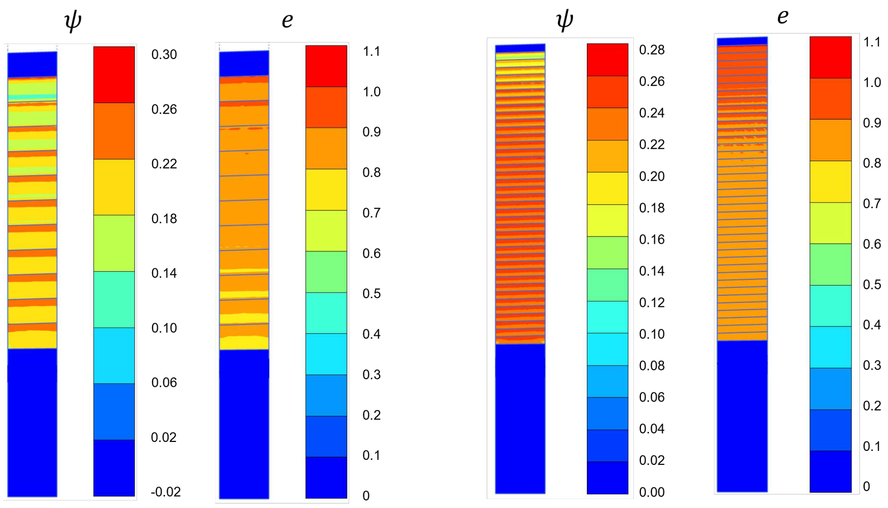

Figure 11: Void ratios and state parameters at the end of deposition ($\psi_0 = 0.1$). (a) Coarse layers, (b) Fine layers

5 CONCLUSIONS

A simplified procedure to analyze the stability of tailings storage facilities (TSFs) employing a quasi-1D soil column was proposed. The NorSand constitutive model was used for the analyses, and the importance of the calibration of the material parameters was discussed. A fully automated script was developed to produce meshes for various geometries and material parameters. The entire deposition sequence was modeled, and a push over undrained analysis was carried out in each case. The proposed strategy reproduces the main features of tailings behavior along the described stress path, estimates the brittleness of tailings at various depths, and produces a screening-level assessment of the risk of static liquefaction of a large TSF. The procedure is expedite enough to allow for performing stochastic analyses to capture the uncertainty of the geometry and material parameters.

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