Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XXXIX, págs. 1021-1026 (artículo completo)
F.E. Dotti, M. Febbo, S.P. Machado, M. Saravia, M.A. Storti (Eds.)
Bahía Blanca, 1-4 Noviembre 2022

OCEAN ENERGY HARVESTING FROM A ROTATING PARAMETRIC PENDULUM WITH BOUNDED CONTROL POWER

Andrés García[a] **and Santiago Salvatierra**[b]

[a]*Grupo de Investigación en Multifísica Aplicada, Universidad Tecnológica Nacional, Facultad Regional Bahía Blanca, 11 de Abril 461, Bahía Blanca, Buenos Aires, Argentina, andresgarcia@frbb.utn.edu.ar*

[b]*Departamento de Ingeniería Eléctrica, Universidad Tecnológica Nacional, Facultad Regional Bahía Blanca, 11 de Abril 461, Bahía Blanca, Buenos Aires, Argentina*

Keywords: Parametric pendulum, Singular optimal control, Energy harvesting, State-constraints

Abstract. Recently, energy harvesting from sea waves was studied using a controlled rotating pendulum. Consisting of a pendulum with a vertical motion induced by the sea waves, pendulum's rotations generate energy to be extracted by an electrical generator attached to its axis.

Following this ideas, a complete dynamic model in state-space variables can be written, with the control input given by the torque of a brush-less dc motor and the pendulum's rotational energy is the object to be controlled. On the other hand, an optimal setup for this control problem is defined, so a user tuned function acts as a set-point (desired pendulum's trajectory).

Moreover, the harvested energy comes from the generator attached to the rotating pendulum, whereas the DC motor must be fed to provide the necessary control action. In this analysis, it is rather important to keep track the balance: output power vs input control power.

To account for this balance, in this paper, besides the optimal control horizon in order to tune the rotating energy, an additional constraint is added to limit the amount of power used by the DC motor, ensuring a real energy harvesting.

The Pontryagin's principle can be used to solve this singular optimal control problem resulting in a classic sign function. This feedback control requires only angular position and velocity's measurements, allowing this controller to be easily implemented in hardware.

Implementing the obtained control law along with the complete mechanical model in Matlab/Simulink, some simulations will show the effectiveness of the energy harvesting principle. The tracking properties and effective harvesting form ocean sea waves will be shown.

1 INTRODUCTION

Sea waves represents a very promising energy source in helping the everyday increasing energy demand worldwide (see for instance (Liu et al., 2020) and (Scruggs and Jacob, 2009)). Nowadays several projects have proposed the use of control devices to extract the mechanical ocean energy power(see (Cheung and F, 2007) and (Zhu et al., 2022)).

Pioneered by Wiercigroch a pendulum system would be feasible for such an energy harvesting purpose pointing toward to an optimal extraction (see for instance(Wiercigroch, 2010) and (Dotti et al., 2017)).

These devices consist of a pendulum with a vertical motion induced by the sea waves. As it is well known, pendulum's stable rotations generate enough energy able to be extracted by an electrical generator attached to its axis (Dotti et al., 2017).

In a recent paper (Garcia et al., 2020), controlled energy harvesting from sea waves was studied for a rotating pendulum. By using Pontryagin's optimal principle with a singular cost functional defined, a closed-loop control law was derived as shown in Eq. (1) (see (Garcia et al., 2020) and (Geering, 2007)):

$$min_{u\in U} \quad \frac{1}{2}\left(\dot{\theta}(t)-\phi(t)\right)^2 \tag{1}$$

such that:

$$\dot{X}(t)=\begin{bmatrix} x_2(t) \\ h(x_1,x_2,t)\end{bmatrix}+\begin{bmatrix}0\\-1\end{bmatrix}\cdot u(t)$$

where $u(t)$ is the control input torque provided by a brush-less dc motor,$E(t)=\frac{1}{2}\cdot m\cdot L^2\cdot \dot{\theta}(t)^2$ is the pendulum's rotational energy and m, L the pendulum's mass and length respectively, with ϕ a user-defined function to set the desired pendulum's behaviour to harvest the ocean energy (in general is a constant rotation).

This singular optimal control technique have demonstrated to be very effective and cost efficient (see for instance Monte et al. (2018) and García and Pons (2017)). However, even when the same framework can account for input and state constraints, a real energy harvesting system is usefulness unless the output power is ensured to be bigger than the control power: $u(t)*\dot{\theta}(t)\leq\frac{dE(t)}{dt}$.

In this paper, taking into account the non-linear state-space model along with the aforementioned input-state constraint (to account for a real energy harvesting), a simple yet effective control law will be extended from the one in (Garcia et al., 2020) using Pontryagin's principle.

The resulting optimal policy and the resulting controller will allow several possibilities, beside the main objective in this paper: stable controlled rotations.

This paper is organized as follows: Section 2 present the non-linear state-space model with the singular optimal control problem and the controller obtained, Section 3 shows some Matlab/Simulink simulations exhibiting the possibility of being tuned for stable/unstable rotations or even asymptotic stability and finally, Section 4 discuss the results and future work.

2 MODELLING AND CONTROL FORMULATION

Following the neat and simplified model in (Dotti et al., 2017), a second order ordinary differential equation can be written in Eq. (2):

$$\ddot{\theta}(t) + \beta \cdot \dot{\theta}(t) + \left(R \cdot cos(\omega \cdot t) + \lambda \cdot R \cdot \frac{\Lambda_3}{\Lambda_1^3} + \lambda \cdot \frac{\Lambda_2}{\Lambda_1} + 1 \right) \cdot sin(\theta(t)) + u(t) = 0 \tag{2}$$

Where $\lambda_1 =$, $\Lambda_2 =$ and $\Lambda_3 =$ and ω a constant. Moreover, the model can be recast in state-space form adding the power constraint, leading Eq. (3):

$$min_{u \in U} \quad \frac{1}{2} \left(\dot{\theta}(t) - \phi(t) \right)^2$$

such that:

$$\begin{cases} \dot{X}(t) = \begin{bmatrix} x_2(t) \\ -\omega \cdot x_1 \\ x_4 \\ h(x_1, x_2, x_3, x_4) \end{bmatrix} + \begin{bmatrix} 0 \\ -1 \end{bmatrix} \cdot u(t) \\ u(t) * \dot{\theta}(t) \leq \frac{dE(t)}{dt} \end{cases}$$

Where $x_3 = \theta(t), x_4(t) = \dot{\theta}(t)$, $\{x_1, x_2\}$ artifitial variables to write in state-space form the function $cos(\omega \cdot t)$, $h(x_1, x_2, t) = -\beta \cdot x_1 - \left(R \cdot x_1 + \lambda^3 \cdot R \cdot \frac{\Lambda_3}{\Lambda_1^3} + \lambda \cdot R \cdot \frac{\Lambda_2}{\Lambda_1} + 1 \right) \cdot \frac{x_2}{\omega}$ and $\phi(t)$ acts as a set-point (desired pendulum's trajectory). Notice that tuning the function $\phi(t)$, different controlled behaviours for the pendulum's orbits can be achieved.

Then, Pontryagin's principle solve this singular optimal control problem, providing a very simple controller in Eq. (3) (see Geering (2007), pp. 49-51):

$$\begin{cases} u(t) = -K \cdot sign\left(\dot{\theta}(t) - \phi(t) \right), & u(t) * \dot{\theta}(t) < \frac{dE(t)}{dt} \\ u(t) = m \cdot L^2 \cdot \frac{h(\theta, \dot{\theta})}{1 + m \cdot L^2}, & u(t) * \dot{\theta}(t) = \frac{dE(t)}{dt} \end{cases} \tag{3}$$

with $K \in \Re^+$ and arbitrary constant and $sign(.)$ the classic sign function.

It turns out that the controller is very simple, even when the complete state-space is needed for feedback: only sign is needed using Arduino and analog to digital conversions (see Figure Fig. 1).

3 MATLAB/SIMULINK SIMULATIONS

Matlab/Simulink simulations shows an very tight follow from $\dot{\theta}(t)$ to $\phi(t)$ and also the effectiveness in harvesting energy (positive difference $\dot{E}(t) - u(t) \cdot \dot{\theta}$, as shown in Figures 2 and 3.

It is worth to notice that, without the additional control constraint on this paper, the harvest energy is not necessarily ensured to be bigger than the control energy as shown in Figure 4, where the control constraint has been removed.

4 CONCLUSIONS

In this paper, the problem to control a rotating parametric pendulum to harvest sea wave ocean energy was presented in an optimal way has been presented. Including an additional

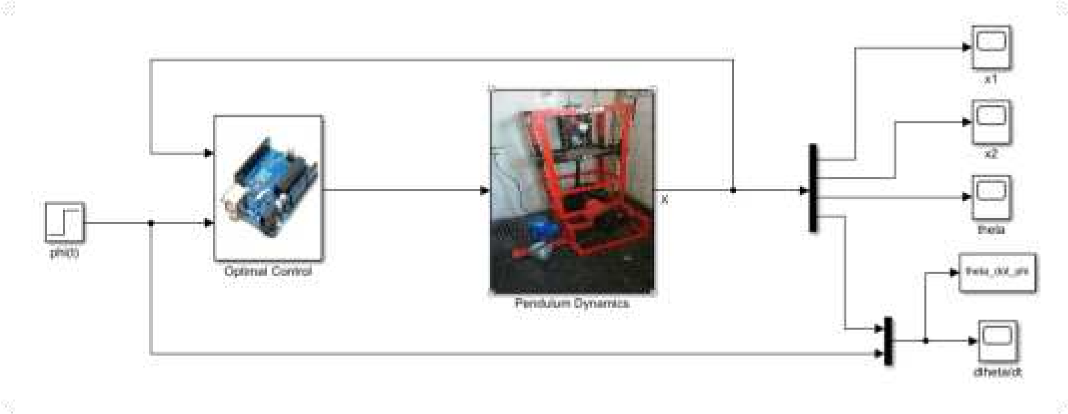

Figure 1: Matlab-Simulink model with singular optimal control and constraints

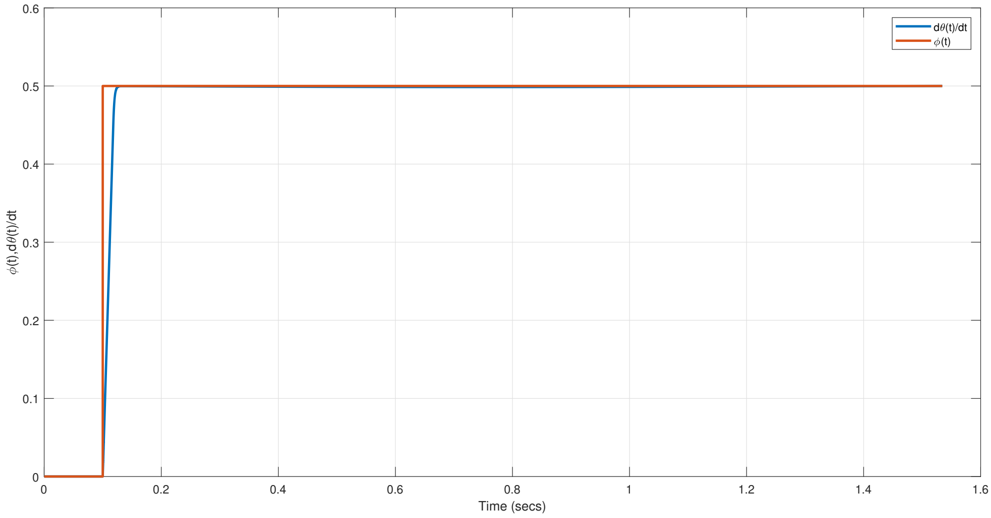

Figure 2: Comparison (following error) between $\phi(t)$ and $\dot{\theta}(t)$

constraint to account for a real energy harvesting: $u(t) * \dot{\theta}(t) \leq \frac{dE(t)}{dt}$, the output energy (harvest) is ensured to be bigger in comparison with the energy used to control the rotations (control power).

Matlab/Simulink simulations using a SimMechancis models plus a non-linear switching control obtained using Pontryagin's principle, have shown the effectiveness of the optimal strategy.

It can be concluded that a fine tuning must be carried out to simulate this non-linear control system with the additional control constraint, so as a future work the truly non-linear closed-loop behaviour will be analysed using asymptotic stability techniques.

ACKNOWLEDGEMENTS

The authors acknowledges to Universidad Tecnológica Nacional and GIMAP.

REFERENCES

Cheung J. and F C. Ocean wave energy harvesting devices. page 34, 2007.
Dotti F., Reguera F., and Machado S. Rotations of the parametric pendulum excited by a recip-

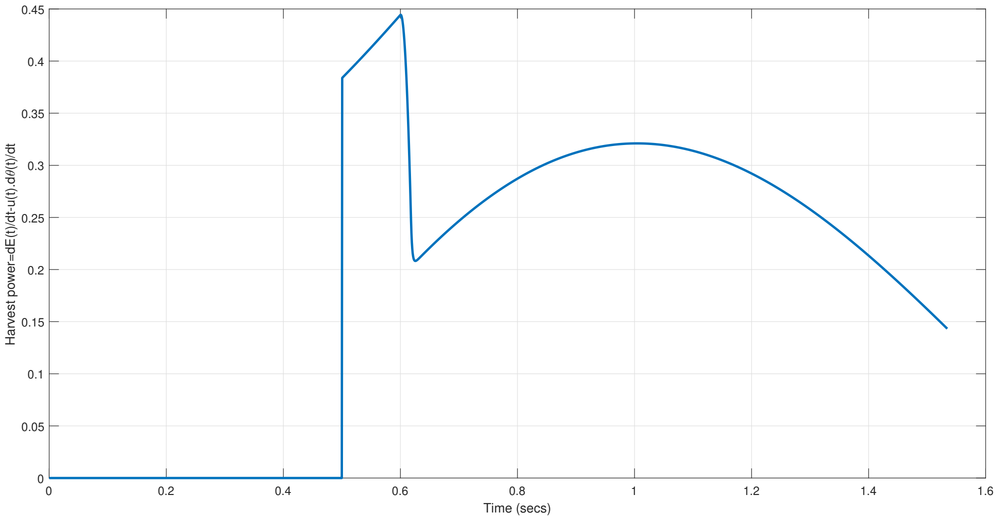

Figure 3: Difference between output power $\dot{E}(t)$ and control power $u(t) \cdot \dot{\theta}(t)$

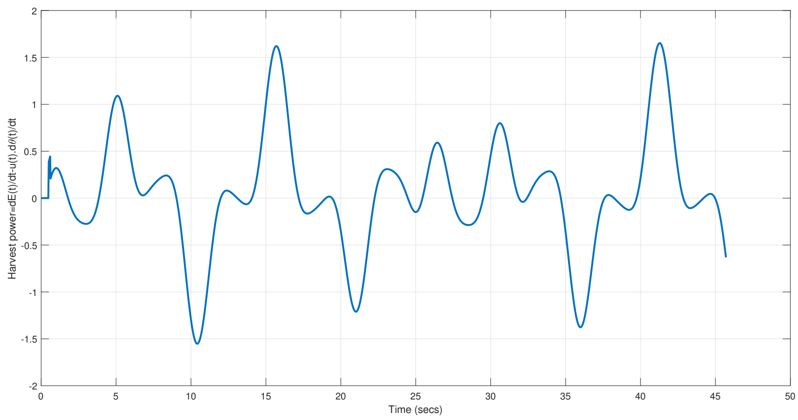

Figure 4: Difference between output power $\dot{E}(t)$ and control power $u(t) \cdot \dot{\theta}(t)$ in the case with no harvest power constraint

rocating motion with a view on energy harvesting. *DINAME 2017. DINAME 2017. Lecture Notes in Mechanical Engineering. Springer*, 2017.

García A. and Pons L. Optimal control for energy harvesting: Electrical model and measurements. *International Journal of Circuits and Electronics*, 2, 2017.

Garcia A., Leonhardt E., Dotti F., and Vera C. Energy harvesting from a rotating parametric pendulum: Singular optimal control. *International Conference on Engineering Vibrations Aberdeen. Scotland*, 2020.

Geering H.P. *Optimal Control with Engineering Applications*. Springer Berlin, Heidelberg, 2007.

Liu Y., Ding B., Zhou B., Cong P., and Zheng S. Editorial: Advances and challenges in ocean wave energy harvesting. *Frontiers in Energy Research*, 8, 2020. ISSN 2296-598X. doi: 10.3389/fenrg.2020.614904.

Monte G., Marasco D., García A., and Perotti E. On-chip spectral analysis with low power and optimal control for energy harvesting using piezoelectric devices. In *IECON 2018 -*

44th Annual Conference of the IEEE Industrial Electronics Society, pages 3858–3863. 2018. doi:10.1109/IECON.2018.8591488.

Scruggs J. and Jacob P. Harvesting ocean wave energy. *Science*, 323(5918):1176–1178, 2009. doi:10.1126/science.1168245.

Wiercigroch M. A new concept of energy extraction from waves via parametric pendulor. *UK patent application*, 2010.

Zhu Y., Yang Z., Jiao C., Ma M., and Zhong X. A multi-modal energy harvesting device for multi-directional and low-frequency wave energy. *Frontiers in Materials*, 9, 2022. ISSN 2296-8016. doi:10.3389/fmats.2022.898921.