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COMPARISON OF STABILIZATION SCHEMES TO SIMULATE TWO-PHASE FLOWS AND MISCIBLE DISPLACEMENTS IN POROUS MEDIUM BY FINITE ELEMENT

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Abstract. *This work presents a numerical comparison of stabilized finite element formulations to solve slightly compressible immiscible two-phase flows and miscible displacements in porous media. The coupled set of partial differential equations is approximated in space by the Streamline Upwind Petrov-Galerkin (SUPG) or the Algebraic Subgrid-Scale (ASGS) stabilized finite element formulations. Shock-capturing operators are added to the formulations to improve stability around the moving sharp fronts. Both formulations are evaluated numerically regarding their accuracy and computational performance.*

1 INTRODUCTION

Oil recovery from a reservoir can be greatly improved by injection of water or other fluids that can be immiscible or miscible with oil. When simulating such processes a variety of methods have been employed such as finite differences, pseudo-spectral and the modified method of characteristics combined with mixed finite elements. Finite differences are adopted in most commercial reservoir simulators, despite its difficulties to handle complex geometries. Therefore, there is an renewed interest in unstructured grid approaches based on control volumes of finite elements to allow more geometric flexibility. However, one of the main difficulties when simulating numerically these processes is the advective-dominated transport in porous media. Several approaches can be used when solving properly such problems. Classical numerical methods either lacks stability, resulting nonphysical instabilities, or accuracy, when excessive numerical diffusion is produced. Despite the attention to this fundamental problem has received in the past decades, there is still need for better numerical techniques[1].

Here we adopt a classical Galerkin method to compute pressure and a post-processing technique[2] to evaluate velocities with equal interpolation order. When solving the advection dominated saturation/concentration equation, we are interested in stability and accuracy. Stabilized finite element methods are particularly interesting for those cases. We employ two distinct stabilized methods to solve the saturation/concentration equation. One is the Streamline Upwind Petrov-Galerkin (SUPG), developed by Brooks and Hughes[3] to control spurious numerical oscillations plus a discontinuity capturing technique known as Consistent Approximated Upwind (CAU) developed by Galeão and do Carmo[4]. The other is a Multiple Scale formulation as described in Juanes and Patzek[1] with an Algebraic approximation to the Subgrid-Scale (ASGS) plus a subscale-driven artificial diffusion to capture shocks near sharp fronts.

In this work we compare numerically the SUPG plus the CAU discontinuity capturing formulations with the ASGS plus the subscale-driven discontinuity operator in order to evaluate good and bad aspects concerning both formulations.

The remainder of this paper is organized as follows. In the next section we briefly review the immiscible and miscible flow mathematical formulations. Section 3 presents the SUPG and the ASGS finite methods for both cases, with velocity post processing and time integration remarks. Two numerical examples are studied to examine robustness, accuracy and behavior of both formulations. One is the immiscible two-phase flow in a five spot configuration and is a miscible displacement flow in a homogeneous channel with nonmonotonic viscosity profile. The paper ends with the summary of main conclusions observed.

2 GOVERNING EQUATIONS

2.1 Immiscible displacement

The mathematical model of the immiscible displacement of two slightly compressible

fluids through a rigid porous medium can be described by a system of partial differential equations[5]. These equations, in a domain $\Omega \in R^2$ with a contour Γ at a time interval [0, T], can be written as

$$\phi c_t \frac{\partial p}{\partial t} + \nabla \cdot \mathbf{v}_T = Q_t \text{ on } \Omega \times [0, T] \tag{1}$$

$$\mathbf{v}_T = -\mathbf{\Lambda}_p \nabla p - \mathbf{\Lambda}_m \frac{dp_c}{ds_w} \nabla s_w + (\rho_0 \lambda_0 + \rho_w \lambda_w)\mathbf{K}\mathbf{g} \text{ on } \Omega \times [0, T] \tag{2}$$

$$\phi \frac{\partial s_w}{\partial t} + \nabla \cdot (\mathbf{v}_a + \mathbf{D} \nabla s_w) + Q_w = 0 \text{ on } \Omega \times [0, T] \tag{3}$$

where the subscripts w and o refer to the water and oil phases respectively. In eq. (1) p is the average pressure of the water and oil phases, ϕ is porosity, c_t is the total compressibility of rock-fluid system, $\mathbf{v}_T = \mathbf{v}_w + \mathbf{v}_o$ is the total velocity of the system and $Q_t = Q_w + Q_o$ is the total volumetric injection rate. In eq. (2) s_w is the saturation of water, p_c is the capillary pressure, ρ_w and ρ_o are the density of water and oil respectively. The phase mobilities of the phases λ_j are given by

$$\lambda_j = \frac{k_{rj}}{\mu_j}, j = w, o \tag{4}$$

where k_{rj} are the relative permeabilities and μ_j is viscosity. $\mathbf{K}$ is the position-dependent absolute permeability tensor and $\mathbf{g} = g\nabla z$ where g is gravity and z is depth. The tensors $\mathbf{\Lambda}_p$, $\mathbf{\Lambda}_m$ are defined as

$$\mathbf{\Lambda}_p = \mathbf{K}(\lambda_0 + \lambda_w) \tag{5}$$

$$\mathbf{\Lambda}_m = \frac{1}{2}\mathbf{K}(\lambda_0 - \lambda_w) \tag{6}$$

In eq. (3) the apparent fluid velocity $\mathbf{v}_a$ is defined as

$$\mathbf{v}_a = f_w[\mathbf{v}_T + \lambda_0(\rho_w - \rho_0)\mathbf{K}\mathbf{g}] \tag{7}$$

The tensor $\mathbf{D}$ is given as

$$\mathbf{D} = \lambda_0 f_w \frac{dp_c}{ds_w}\mathbf{K} \tag{8}$$

where f_w is the fractional flow of water defined as

$$f_w = \frac{\lambda_w}{\lambda_w + \lambda_o} \tag{9}$$

and Q_w is the water injection rate.

We assume proper initial and boundary conditions for the pressure and saturation equations. Please refer to Coutinho et al.[6] for a more detailed presentation.

2.2 Miscible displacement

The mathematical model of the miscible displacement of one incompressible fluid by another, in a rigid porous medium can be described by a system of partial differential equations[5]. These equations in a domain $\Omega \in R^2$ with a contour Γ at a time interval [0, T], can be written as:

$$\nabla \cdot \mathbf{v} = q \ \text{ on } \Omega \times [0, T] \tag{10}$$

$$\mathbf{v} = -\mathbf{A}(c).\nabla p \ \text{ on } \Omega \times [0, T] \tag{11}$$

$$\phi \frac{\partial c}{\partial t} + \nabla \cdot (c\mathbf{v} - \mathbf{D}(\mathbf{v}) \cdot \nabla c) = \hat{c} q \ \text{ on } \Omega \times [0, T] \tag{12}$$

where **v** is the total Darcy velocity, p is the fluid pressure, c is the concentration of the fluid mixture, ϕ is the porosity of the porous medium, and **D**(**v**) is the diffusion-dispersion tensor given as

$$\mathbf{D}(\mathbf{v}) = \frac{1}{|\mathbf{v}|^2} \begin{bmatrix} v_1 & -v_2 \\ v_2 & v_1 \end{bmatrix} \begin{bmatrix} D_{//} & 0 \\ 0 & D_{\perp} \end{bmatrix} \begin{bmatrix} v_1 & v_2 \\ -v_2 & v_1 \end{bmatrix} \tag{13}$$

where v_1, v_2 are the cartesian velocity components, $D_{//}$ and $D\perp$ are respectively the longitudinal and transverse dispersion coefficients. The wells are represented by source and sink terms denoted by q. The function $\hat{c}$ is specified at the sources and is equal to the resident concentration at the sinks. We assume proper initial and boundary conditions for the pressure and concentration equations. Please refer to Coutinho and Alves.[7,8] for a detailed presentation. The tensor **A**(c) is given as

$$\mathbf{A}(c) = \frac{\mathbf{K}}{\mu(c)} \tag{14}$$

where **K** is the position-dependent absolute permeability tensor in porous media flow and μ(c) is the viscosity of the fluid mixture, which is a nonlinear function of the concentration.

3 NUMERICAL FORMULATION

In this section we present the semi-discrete stabilized finite element formulations applied

to the governing equations for immiscible and miscible displacements. The semi-discrete formulation is characterized by a finite element discretization in space followed by a finite difference discretization in time. The SUPG formulation is applied to the saturation and to the concentration equation respectively and the Galerkin formulation is applied to the pressure equation in both of cases.

3.1 Immiscible displacement

We consider the space domain Ω divided in n_{el} elements, Ω_e, e=1, 2,…,n_{el}, where $\Omega=\cup_{e=1}^{nel}\Omega_e$ and $\Omega_i \cap \Omega_j=\emptyset$. Given the spaces for the test functions for the pressure p^h and the water saturation s^h and the space of admissible variations w^h respectively defined as

$$p^h = \{p^h / p^h \in [h^h(\Omega)], p^h / \Omega_e \in [P^1(\Omega^e)], p^h(t) = \overline{p} \text{ in } \Gamma_d\} \tag{15}$$

$$s^h = \{s_w^h / s_w^h \in [h^h(\Omega)], s_w^h / \Omega_e \in [P^1(\Omega^e)], s_w^h(t) = \overline{s}_i^h \text{ in } \Gamma_i\} \tag{16}$$

$$w^h = \{w^h / w^h \in [h^h(\Omega)], w^h / \Omega_e \in [P^1(\Omega^e)], w^h = 0 \text{ in } \Gamma\} \tag{17}$$

where $h^h(\Omega)$ is a space of finite dimension functions over Ω, $P^1(\Omega^e)$ represents polynomials of first order in Ω^e. Considering a standard discretization of Ω into finite elements the Galerkin formulation for the pressure equation is written as

$$\int_\Omega w^h \phi c_t \frac{\partial p}{\partial t} d\Omega - \int_\Omega \nabla w^h \cdot v_T d\Omega = \int_\Omega w^h Q_T d\Omega \tag{18}$$

The weak variational approximation for the saturation equation is written as

$$\begin{aligned} &\int_\Omega w^h (L^h(s_w^h, v_a^h)) d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \tau L^* L^h(s_w^h, v_a^h) d\Omega \\ &+ \sum_{e=1}^{nel} \int_{\Omega^e} \delta(s_w^h) \nabla w^h \nabla s_w^h d\Omega = 0 \end{aligned} \tag{19}$$

where the differential operator $L(s_w^h, v_a^h)$ is given by,

$$L(s_w^h, v_a^h) = \phi \frac{\partial s_w^h}{\partial t} + \nabla \cdot (v_a^h - \mathbf{D} \nabla s_w^h) + Q_w \tag{20}$$

and the differential operator L^* is defined as

$$L^* = v_a^h \cdot \nabla w^h \tag{21}$$

The first integral in eq. (19) is the Galerkin term, the first summation of element-level integrals is the advection stabilization term and the second is the shock-capturing term. The particular forms of stabilization and shock-capturing terms will be defined later.

3.2 Miscible displacement

We consider the space domain Ω divided in n_{el} elements, Ω_e, e=1, 2,…,n_{el}, where $\Omega=\cup_{e=1}^{nel}\Omega_e$ and $\Omega_i \cap \Omega_j=\emptyset$. Given the spaces for the test functions for the pressure p^h and the concetration c^h and the space of admissible variations w^h respectively defined as

$$p^h = \{p^h / p^h \in [h^h(\Omega)], p^h / \Omega_e \in [P^1(\Omega^e)], p^h(t) = \overline{p} \text{ in } \Gamma_d\} \tag{22}$$

$$c^h = \{c^h / c^h \in [h^h(\Omega)], c^h / \Omega_e \in [P^1(\Omega^e)], c^h(t) = \overline{c}^h \text{ in } \Gamma_i\} \tag{23}$$

$$w^h = \{w^h / w^h \in [h^h(\Omega)], w^h / \Omega_e \in [P^1(\Omega^e)], w^h = 0 \text{ in } \Gamma\} \tag{24}$$

where $h^h(\Omega)$ is a space of finite dimension functions over Ω, $P^1(\Omega^e)$ represents polynomials of first order in Ω^e. Considering a standard discretization of Ω into finite elements the Galerkin formulation for the pressure equation is written as

$$\int_\Omega \nabla w^h \cdot \mathbf{A}(c^h) \nabla p^h d\Omega = \int_\Omega w^h q d\Omega \tag{25}$$

The weak variational approximation for the concentration equation is written as

$$\begin{aligned} &\int_\Omega w_c^h L(c^h) d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \tau L^* R(c^h) d\Omega \\ &+ \sum_{e=1}^{nel} \int_{\Omega^e} \delta(c^h) \nabla w_c^h \nabla c^h d\Omega = \int_\Omega w_c^h \hat{c}^h q d\Omega \end{aligned} \tag{26}$$

The differential operator $L(c^h)$ is given by,

$$L(c^h) = \phi \frac{\partial c^h}{\partial t} + \nabla \cdot (c^h \mathbf{v}_p^h - \mathbf{D}(\mathbf{v}_p^h) \nabla c^h) \tag{27}$$

and the differential operator L^* is given by,

$$L^* = \mathbf{v}_p^h \cdot \nabla w_c^h \tag{28}$$

The discrete residual is defined as

$$R(c^h) = L(c^h) - \hat{c}^h q \tag{29}$$

Similarly to the immiscible case, the first integral in eq. (26) is the Galerkin term, the first summation of element-level integrals is the convection stabilization term and the second is the shock-capturing term, needed to add stability around the moving sharp concentration fronts. The particular form of these operatores will be defined below.

3.3 SUPG and ASGS stabilized formulations

The SUPG and the ASGS formulations for immiscible and miscible displacements are very

similar. In the case of advection-dominated flows with no gravity effects and using linear elements we have:

Immiscible case

$$\text{SUPG: } L^* w^h = v_a^h \cdot \nabla w^h \text{ and } \tau = \left(c_2 \frac{\left\| v_a^h(s_w^h) \right\|}{h} \right)^{-1} \tag{30}$$

$$\text{ASGS}^1\text{: } L^* w^h = va_a^h(s_w^h) \cdot \nabla w^h \text{ where } va_a^h(s_w^h) = v_a^h + \frac{\mathrm{d}\mathbf{D}}{\mathrm{d}s_w} \nabla s_w^h \tag{31}$$

$$\text{and } \tau = \left(c_1 \frac{\left\| \mathbf{D}(s_w^h) \right\|}{h^2} + c_2 \frac{\left\| va_a^h(s_w^h) \right\|}{h} \right)^{-1} \tag{32}$$

Miscible case:

$$\text{SUPG: } L^* w_c^h = \mathbf{v}_p^h \cdot \nabla w_c^h \text{ and } \tau = \left(c_2 \frac{\left\| \mathbf{v}_a^h \right\|}{h} \right)^{-1} \tag{33}$$

$$\text{ASGS}^1\text{: } L^* w_c^h = \mathbf{v}_p^h \cdot \nabla w_c^h \text{ and } \tau = \left(c_1 \frac{\left\| \mathbf{D} \right\|}{h^2} + c_2 \frac{\left\| \mathbf{v}_a^h \right\|}{h} \right)^{-1} \tag{34}$$

where h is the element geometric parameter and c_1 and c_2 are constant coefficients.

The shock-capturing capture operators added to the stabilized formulations are very similar. In general they are defined as:

$$\int_{\Omega^e} \delta(u^h) \nabla w^h \nabla u^h d\Omega \tag{35}$$

where u is the transported quantity, and δ is a nonlinear diffusion parameter. For the SUPG formulation we add the CAU shock-capturing, introduced by Galeao and do Carmo[4], which has the form:

$$\delta = \alpha h \frac{\left| R(u^h) \right|}{\left\| \nabla u^h \right\|} \tag{36}$$

where α is a parameter that depends on the Peclet number. Note that in CAU operator, the shock-capturing term is nil when $\left\| \nabla u^h \right\|$ is zero. For the ASGS formulation the shock-capturing parameter has the form,

$$\delta = C_{sc} \frac{\left|\tau R(u^h)\right|}{U_{sc}} h \left|\mathbf{v} a_a^h(u_h)\right| \tag{37}$$

where C_{sc} is a constant coefficient and U_{sc} is a characteristic value of the solution near the shock.

3.4 Velocity Post-processing

In the case of immiscible as well as miscible displacements the velocity computed directly from Darcy's law is less accurate than the other variables. Post-processing schemes may be used to obtain satisfactory results. Here we adopted the global post-processing scheme of Malta et al.[2]. This scheme is based in the combination of Darcy's law variational formulation and the residue of the mass conservation equation. Given the pressure p^h and the saturation s^h (or concentration c^h) and defining

$$U^h \in \{\mathbf{w}^h \in H^h(\Omega) \times H^h(\Omega), \mathbf{w}^h \cdot \mathbf{n} = 0 \text{ in } \Gamma_n\} \tag{38}$$

the velocity post-processing consist in finding $\tilde{\mathbf{v}}_t^h \in U^h$ such as $\forall \mathbf{w}^h \in U^h$ we have

$$\int_\Omega \mathbf{w}^h \cdot (\mathbf{B}_1 \tilde{\mathbf{v}}_t^h + \mathbf{B}_2 \nabla p_n^h + \mathbf{B}_3) d\Omega + \sum_{e=1}^{nel} \int_{\Omega^e} \sigma \nabla \cdot \mathbf{w}^h (\nabla \cdot \tilde{\mathbf{v}}_t^h - q) d\Omega^e = 0 \tag{39}$$

where $\tilde{\mathbf{v}}_t^h$ is the post-processed velocity vector and the parameter σ is a mesh geometry parameter. In the miscible case $\mathbf{B}_1 = \mathbf{I}$, $\mathbf{B}_2 = \mathbf{K}\lambda_t$, $\mathbf{B}_3 = -\mathbf{K}\lambda_w \frac{dp_c}{ds_w} \nabla s_w^h - \mathbf{K}\mathbf{g}(\lambda_w \rho_w + \lambda_n \rho_n)$ while in the immiscible case $\mathbf{B}_1 = \mathbf{A}^{-1}$, $\mathbf{B}_2 = \mathbf{I}$, $\mathbf{B}_3 = \mathbf{0}$. Using this technique the problem variables pressure, velocity and saturation (concentration) are approximated by equal order interpolations.

3.5 Time-marching algorithm

The saturation and the concentration time-derivatives for immiscible or miscible displacements are approximated by the generalized trapezoidal rule (Hughes[9]).Thus, we obtain the following block-iterative predictor-multicorrector algorithm

- Block 1: Solve Pressure Equation
- Block 2: Compute Velocity field
- Block 3: Solve Saturation or Concentration Equation
- Perform the Updates

The iterative process continues up to some convergence criteria are satisfied. In this algorithm the linear system of equations corresponding to the pressure equation is solved by preconditioned conjugate gradients while that corresponding to the saturation (or

concentration) equation is solved by the preconditioned GMRES algorithm. An element-by-element Gauss Seidel preconditioner is used in both of cases. The systems of equations corresponding to the velocity post-processing is solved using simple Jacobi iterations. Here we use variable time step due to the strong nonlinear coupling between the pressure and saturation or concentration equations. We use an automatic time-step selection strategy based on the feedback control theory as presented in Coutinho and Alves[7,8].

4 NUMERICAL RESULTS

4.1 Immiscible case

This numerical example intends to compare both stabilized formulations by solving a five spot problem described in Durlofsky[10]. The computational domain is a 1 by 1 square with 20x20 cells, each subdivided into 2 triangles, generating the finite element mesh shown in Figure 1a. Material properties of the porous media are porosity 0.2 and permeabilities kx = ky = 1 and kxy = 0. Oil and water viscosities are 4 and 1 respectively. Gravitational, capillary pressure and dispersion/diffusion effects are neglected. Initial conditions and constant in time boundary conditions for the pressure equation are given in Figure 1b and for the water saturation equation in Figure 1c. Initial conditions are P=0 for the pressure equation and a oil saturated reservoir for the saturation equation. A fixed time step of 0.005 PVI (pore volume injected) and a total simulation time of 4 PVI were adopted. Actually, the injected pore volume corresponds to a dimensionless time unit[10]. At time 0.8, which corresponds to 1 PVI a steady state solution is achieved, although more time steps were used to asses the behavior and vanishing effects of both formulations.

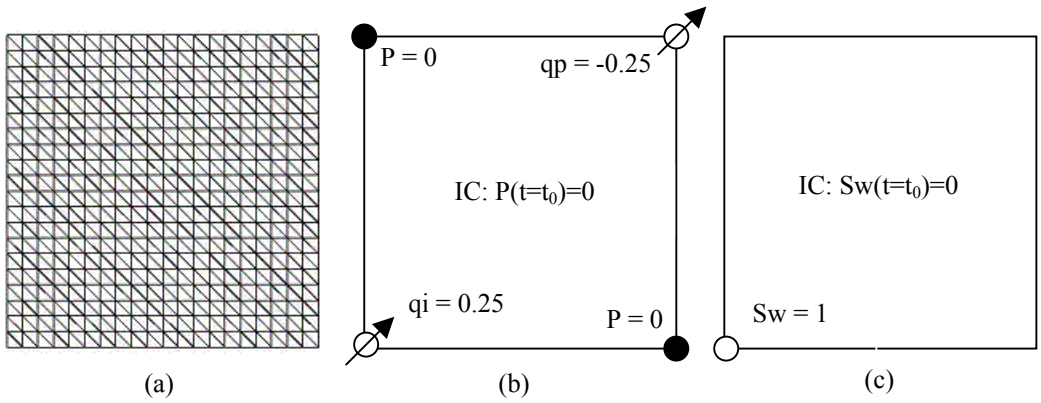

Figure 1: Finite element mesh(a); Boundary and initial conditions for Pressure(b) and Water Saturation(c).

Numerical solutions for water saturation(a), SUPG_CAU(b) and ASGS(c) shock capturing terms per element are presented in Figure 3 for several time steps. By analyzing those results, one can verify that the ASGS follows very well the saturation fronts while the CAU formulation is a bit more dispersive. It is also observed that both formulations vanish after a

steady state solution pattern is attained (i.e. 1 PVI). On the other hand, when having a further insight its clear that the CAU formulation does provide a smoother solution at the sharp front, while the ASGS technique looks very similar to the SUPG without shock capturing stabilization. Actually, as diffusion was neglected, the ASGS intrinsic time parameter is the same as the SUPG stabilization parameter. By that we can conclude that the CAU shock capturing is providing a smoother solution treatment in a proper manner the discontinuity effects at fronts. Those observations were made by analyzing the graph presented in Figure 2, which is the oil saturation at the producing well with PVI dimensionless time unit. Similar tests comparing different choices for the SUPG stabilization term were driven in the work of Coutinho et al.[6] solving the same five spot problem. It is known that in its definition the ASGS formulation has a controlling constant parameter *Csc*. When taking such constant as 1 a poor stabilization was obtained. When using *Csc*=5 a better stabilization was observed, but still not as smooth as the CAU stabilization, as can be observed in Figure 2. We consider that the need for a constant makes the ASGS approach not very robust for practical simulations, what does not happen with the CAU operator. For this example, both approaches had a very similar behavior, taking only 2-3 iterations to achieve non linear convergence (with 0.05 tolerance), as shown in Figure 4.

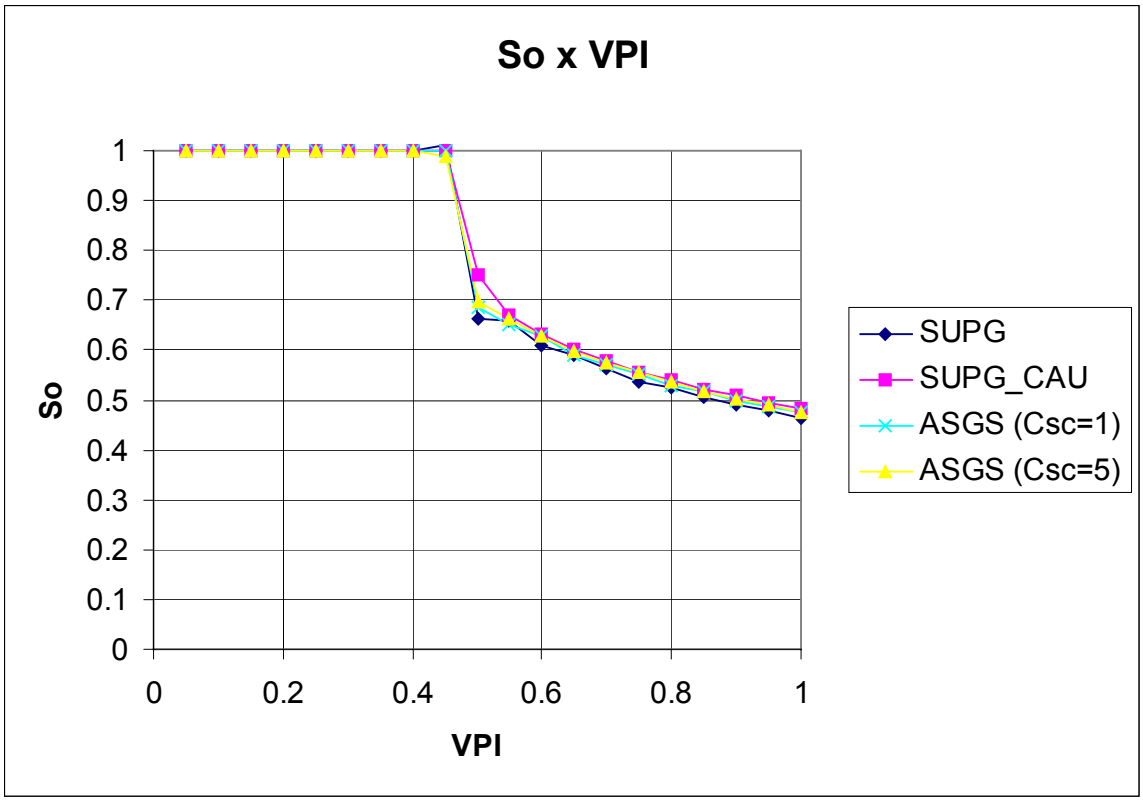

Figure 2: Comparison of métodos for oil saturation at the producer well.

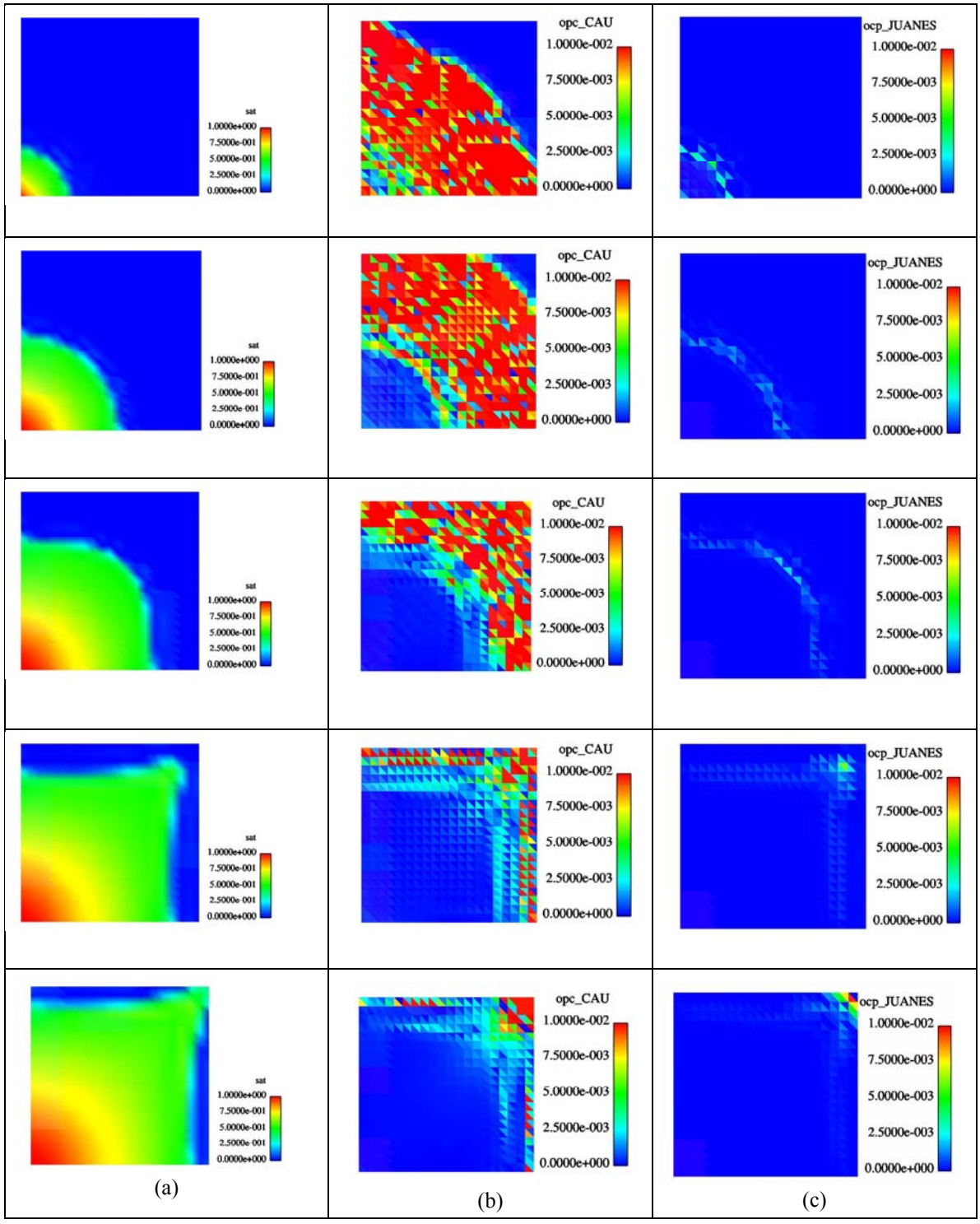

Figure 3: Sw(a), SUPG_CAU(b) and ASGS(c) parameters for time steps 1, 30, 60, 90 and 120.

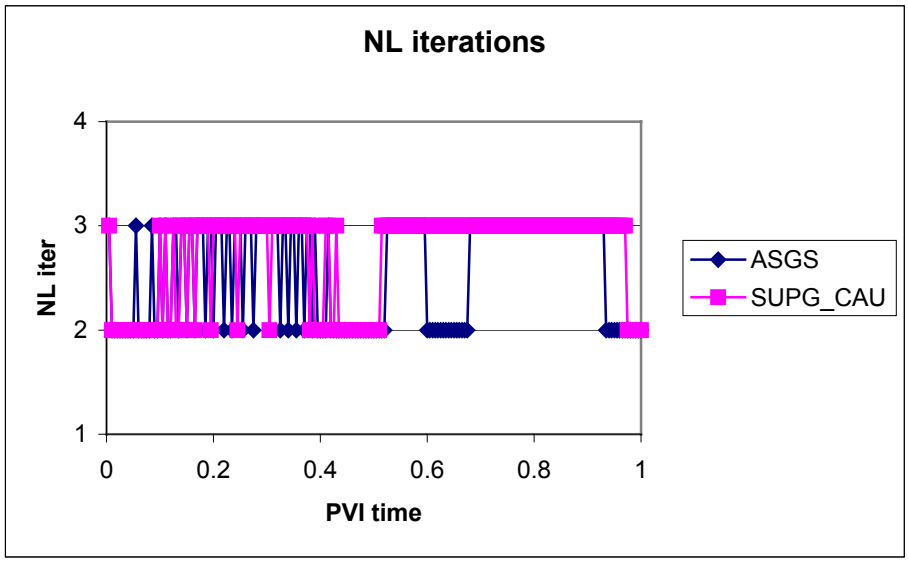

Figure 4: Non linear iterations within each time step: for 1PVI.

4.2 Miscible case

We analyze a miscible displacement flow with nonmonotonic viscosity law as investigated by Manickam and Homsy[11] and also by Coutinho and Alves[8]. The resulting governing equations were scaled in the same manner as in the previous works[11,8] Velocity, length, time and viscosity are scaled with U, D/U, D/U^2 and μ_1, respectively. The diffusion is of the form $\mathbf{D} = D\mathbf{I}$ and D is assumed unitary. The nonmonotonic viscosity law relationships are also given in Manickam and Homsy[11] and Coutinho and Alves[8]. Here, the same computational domain, mesh, initial and boundary conditions were employed. The maximum simulation time was 3100 time units for a fixed time step of 0.5. Although adaptive time stepping based on a PID stepsize control algorithm was already implemented, a fixed time step approach was adopted for comparison of the stabilization formulations. Figure 5 shows the finite element mesh and a detail of the solution at the final time of the analysis.

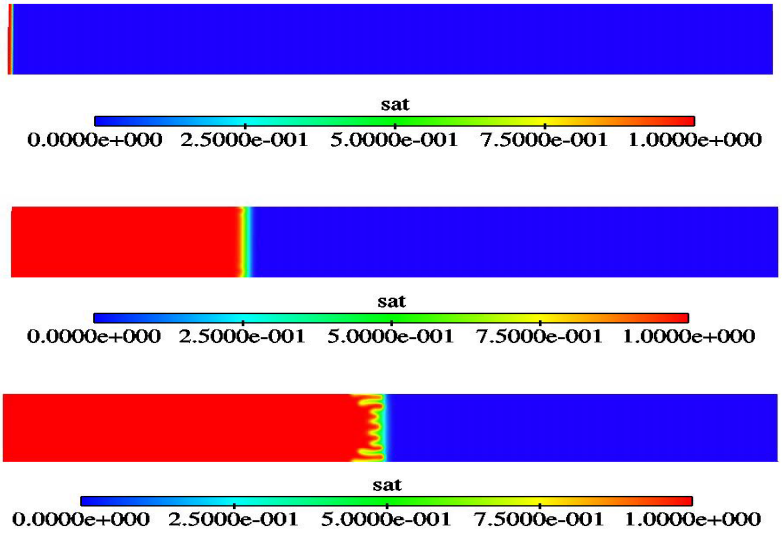

Figure 5: Numerical solution for time steps 1, 1600 and 3200.

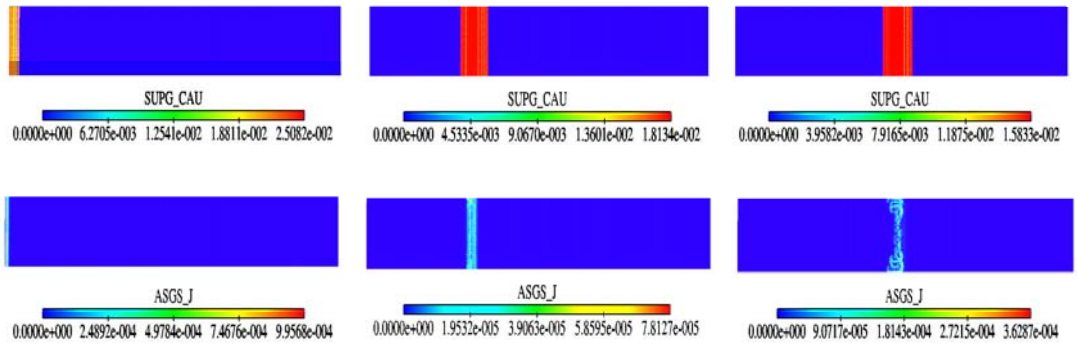

Figure 6: Comparison of shock capturing stabilizations.

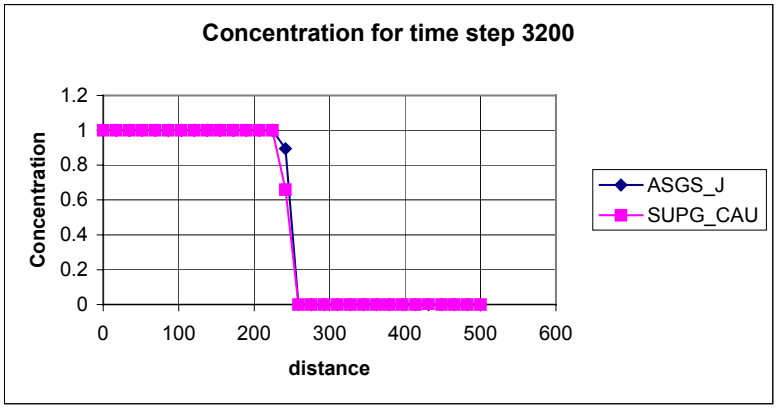

Figure 7: Concentration along the channel for time step 3200.

Numerical solutions for concentration, SUPG_CAU and ASGS shock capturing terms per element are presented in Figure 5 and 6 respectively for time steps 1, 1600 and 3200. By comparing the results for both cases, it is once again noticed that the ASGS plus the subscale-driven discontinuity capturing follows very well the saturation fronts while the CAU formulation is a bit more diffusive. It is also observed that the ASGS shock capturing operator numerical value is smaller than the CAU. This can be adjusted by the *Csc* parameter.

Figure 7 shows the concentration along the channel for time step 3200. As a whole, both stabilizations had a good stabilization of the discontinuity and a good agreement on the number of non linear and solver iterations.

5 CONCLUSIONS

In this paper we presented a numerical comparison between two distinct numerical approaches to the saturation equation, when solving two-phase immiscible porous flow and miscible displacement problems.

The numerical results for the immiscible case points out that the shock capturing terms in both formulations have significant values at the saturation fronts and null values elsewhere. We have observed by numerical experiments that the CAU operator leads to more robust, consistent and smoother solutions at the saturation fronts. Moreover, it does not need control parameters to improve the stabilization, which makes it more practical to be used. Both required a very similar number of non linear iterations. That shows that several techniques and studies exists and could be used with caution to their good and bad properties.

For the miscible case similar results were obtained. Both formulations have a similar behavior, distinguishing only in the amount of stabilization, that was more for the CAU than the other. Despite that, as a preliminary analysis both seemed to achieve almost the same numerical result.

6 ACKNOWLEDGES

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