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COMPUTATIONAL BUCKLING ANALYSIS OF SHELLS: THEORIES AND PRACTICE

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Abstract. *Shell buckling problems belong to the class of geometrically nonlinear behavior, and may be coupled with material nonlinearity of the shell. There are many general-purpose finite element programs that perform geometric and material nonlinear analysis of shells; however, this does not mean that a user can feed data and collect reliable results without a full understanding of the physics of the problem. This paper discusses the theories involved in the explanation and classification of phenomena, and in the prediction of results. Next, those approaches are considered in the practical analysis of one shell form, namely thin-walled steel tanks used to store oil. Results have been obtained with the general-purpose package ABAQUS, and they tend to show that their interpretation requires the use of Koiter's theory in order to make sense of what is obtained. Some thoughts on possible ways to implement Croll's lower bound reduced energy approach in practice are given.*

1. INTRODUCTION

In this paper we consider the phenomenon of buckling as a process of change in the shape of a structure. Engineers design structures with a well-defined shape thoughtfully chosen to fulfill some purpose. But under certain critical conditions the structure cannot withstand further load with the same shape and changes its shape in a slow or sometimes in a violent way. The view of buckling as the development of a new form is relatively recent and has emerged as a consequence of the work of W. T. Koiter [24]. This introduction attempts to establish **what** needs to be computed in a buckling analysis.

Buckling belongs to a class of problems in mechanics for which some form of singularity occurs in an equilibrium situation. Other problems of the same class may be the phase changes in matter, the propagation of cracks, and many others. What such problems have in common is that they are described in terms of control parameters (such as the external loads) and response parameters (such as the displacements), and that at some configuration of the above parameters the system reaches a singularity and enters into a new set of possibilities, not available at the beginning of the process. The singularity is usually called "critical state" and the states beyond it are identified as "post-critical states".

At present, the usual representation of buckling problems is made in terms of equilibrium paths, i.e. a graph showing the evolution of the structural system in a space defined by the control and the response variables. Essentially, one needs to identify a state that separates two regimes, and to compute equilibrium states of both regimes. First, a pre-critical (or fundamental, or primary) equilibrium path is required, often a linear equilibrium problem. Second, the critical state needs to be identified. Third, a post-critical (or secondary) equilibrium path is computed: this is a nonlinear path and often displays an evolution, i.e. further changes in the shape of the structure occur along its way (mode jumping).

In this paper we argue that there are several levels of theories involved in the computational buckling analysis of shells [a]. There are (a) **predictive** theories at the level of computations, in which results are obtained; (b) **explanatory** theories, in which results make sense (or not); and (c) **classification** theories, in which possible solutions are classified. Computational mechanics tends to be associated only to predictive theories, but it will be shown that explanations and classifications are as essential as predictions in any adequate analysis. Three theories are examined: Koiter's initial post-critical behavior [24]; Croll's reduced energy approach [6]; and non-linear computational mechanics. The complementarity of these approaches is illustrated with reference to the buckling analysis of thin-walled steel tanks employed to store liquids.

2. BRIEF REVIEW OF RELEVANT THEORIES

2.1 Koiter's initial post-critical theory

Koiter [24] developed both a theoretical framework for the understanding and classification of the mechanics of buckling, and also a tool to carry out the computations (for a recent review of the developments of the theory, see [12,13]). But while it has almost no competitors at present to understand the process of buckling, only in a few cases a tool has been implemented to perform the computations [7, 8, 9, 22]. In this paper we consider three roles of the

[a] In looking at the different theories used for the solution of a problem, Lakatos [23] stressed the need to use a pluralist model: "In this pluralist model, the conflict does not arise between 'theory and facts', but between two high level theories: an interpretative theory that provides the facts and an explanatory theory that explains them. The interpretative theory may be of a level as high as that of the explanatory theory… The problem is about deciding which theory will be considered as an interpretative theory and which one as an explanatory theory that tentatively explains them".

theory of Koiter and will attempt to show how other approaches used to perform the computations are influenced at present from Koiter's analysis tools.

The **predictive** side of Koiter's approach relies on perturbation analysis. Once a critical state has been detected (meaning that the displacement degrees-of-freedom Q_i^c, the critical load λ^c, and the mode shape x_i have been evaluated) then the post-critical equilibrium path $[\lambda(s), Q_i(s)]$ is computed in terms of a perturbation parameter *s* as:

$$Q_i(s) = Q_i^c + q_i^{(1)} s + ½\, q_i^{(2)} s^2 + \dots \tag{1}$$

$$\lambda(s) = \lambda^c + \lambda^{(1)} s + ½\, \lambda^{(2)} s^2 + \dots \tag{2}$$

where the notation $\lambda^{(j)}$ refers to the j-th derivative of the variable λ with respect to the perturbation parameter. The perturbation equations of the equilibrium conditions are used to evaluate the unknowns $q_i^{(j)}$ and $\lambda^{(j)}$. The accuracy of the solution may be improved by including more terms in the perturbation expansions (1-2); however this accuracy is limited by the order of the energy V in terms of the Q_i.

The main limitation of Koiter's analysis is the quality of the quantitative predictions that can be obtained from perturbation analysis: for shells, the results are only accurate up to displacement amplitudes less than the wall-thickness. Thus, a perturbation analysis can only carry out computations close to the critical state, and as such gives information limited to the initial post-critical behavior of the shell.

As a theory of **classification**, Koiter predicts well all possible solutions that emerge from a critical state. To achieve this, Koiter uses the total potential energy of the system, V. For equilibrium, the condition of stationary energy with respect to the displacement degrees-of-freedom Q_i of a discrete system is stated as [b]:

$$V_i = \partial V / \partial Q_i = 0 \tag{3}$$

Given a small perturbation q_j, equilibrium states for which $V_{ij}\, q_j > 0$ are said to be stable, while a critical state is given by $V_{ij}\, q_j = 0$ in which case $q_j = x_j$ is identified as a critical mode. Thus, a critical state is characterized by

$$V_{ij}\, x_j = 0 \tag{4}$$

The nature of the critical state is given by the scalar quantity $V'_i\, x_i$, where $(\,)' = \partial V / \partial \lambda$.

For $V'_i\, x_i = 0$ → Bifurcation state
$V'_i\, x_i \neq 0$ → Limit state

Bifurcation states can be classified into asymmetric and symmetric behavior by means of the coefficient $C = V_{ijk}\, x_i\, x_j\, x_k$ as

For $C = 0$ → Symmetric Bifurcation
$C \neq 0$ → Asymmetric bifurcation

This is not the end of the road, because symmetric bifurcations can be further classified using the stability coefficient $V_4 = V_{ijkl}\, x_i\, x_j\, x_k\, x_l$ as

For $V_4 > 0$ → Stable symmetric bifurcation
$V_4 < 0$ → Unstable symmetric bifurcation

[b] This is the extension of Koiter's theory to discrete systems, carried out at University College London, since Koiter only worked with continuous systems. However, for computational purposes, the discrete version seems more suitable because the degrees-of-freedom may be those arising from a finite element analysis. See Ref. [12].

The importance of this is enormous: one can classify all possible solutions and be confident that only one of them will occur. The theory also classifies cases in which buckling modes couple at the critical state leading to compound critical states and mode interaction, and this task was completed by Chilver, Thompson, and Supple at University College London and elsewhere in England.

A classification scheme identical to Koiter's but based on dynamic (rather than static) analysis has been formulated by Huseyin [19]. Finally, the only alternative for classification is catastrophe theory, which is more qualitative in nature and allows for multiple load parameter systems, but its success in elastic stability analysis has been very limited.

Koiter's success has been highest regarding its power to **explain** the phenomena of buckling of shells. Here the main point was the discovery that small imperfections (either in the geometry or in the loads) destroy a bifurcation behavior leading to a nonlinear fundamental path with a different critical state. For unstable post-critical behavior this means that there is a reduction in the critical load, and this reduction may be significant for shell problems.

2.2 Croll's reduced energy theory

Perhaps the structural application for which buckling problems are most crucial are shell problems, for which high imperfection-sensitivity has been observed. Croll's approach emerges from the theory of Koiter and uses it as a theoretical framework to obtain approximate values of critical loads and mode shapes for shells. As in the case of Koiter, this is both an explanatory theory and a predictive theory, but it makes no attempt to produce a new classification.

The **explanatory** part of Croll's theory is based on two postulates [6]: (1) Systems that may have changes in their membrane resistance are likely to show significant geometric nonlinear response. On the other hand, systems with changes mainly in their bending resistance do not show highly geometric nonlinear response. (2) Practically significant post-buckling loss of stiffness can only occur when the fundamental equilibrium path contains a significant contribution from membrane energy.

Croll's theory led to the reduced energy approach as a **predictive** tool to carry out the computations. According to this, lower bounds to buckling into a particular mode are given by an analysis in which the membrane energy of the shell is eliminated.

The total potential energy is written in the form

$$V = \frac{1}{2}\int\left(N_{ij}\varepsilon_{ij} + M_{ij}\chi_{ij}\right)dA - \psi \tag{5}$$

for i, j = 1,2; where $N_{ij}\varepsilon_{ij}$ is the membrane energy and $M_{ij}\chi_{ij}$ is the bending energy, and ψ is the load potential. Next, each field is represented in terms of the fundamental path $(\text{-})^F$ in the form

$$\begin{aligned} N_{ij} &= \lambda N^F{}_{ij} + N'_{ij} + N''_{ij}; \varepsilon_{ij} = \lambda\varepsilon^F{}_{ij} + \varepsilon'_{ij} + \varepsilon''_{ij} \\ M_{ij} &= \lambda M^F{}_{ij} + M'_{ij}; \chi_{ij} = \lambda\chi^F{}_{ij} + \chi'_{ij} \end{aligned} \tag{6}$$

where (-)' represents the linear contribution to the variable, while (-)'' is the nonlinear contribution. Substitution of each term in the energy V, and considering stationary energy leads to the condition

$$\delta^2 V = \frac{1}{2}\int [\left(N_{ij}\varepsilon_{ij} + M_{ij}\chi_{ij}\right) + \lambda\left(N^F{}_{ij}\varepsilon''_{ij} + N''_{ij}\varepsilon^F{}_{ij}\right)]dA \tag{7}$$

The first parenthesis leads to the stiffness matrix, while the second one leads to the load-geometry matrix.

Croll and co-workers (see Refs. [2, 27], and others cited in [6]) investigated the contribution of each individual term into the energy, and found those that contributed to stabilize the shell and those that were de-stabilizing. The erosion of a critical state due to the coupling of modes and imperfections could only occur because some of the stabilizing contributions were lost, so that the new approach eliminated those terms from the eigenvalue problem in order to obtain a lower bound to the buckling loads.

The consequences of this are that we may now compute bifurcation buckling of a shell but with a modified version of the total potential energy and obtain good estimates (both quantitative and qualitative) of the true buckling resistance. The modifications consist in the elimination of the membrane energy contribution to the critical state. This does not mean that the membrane energy is eliminated from the fundamental path.

2.3 Nonlinear computational mechanics

There is a whole branch of mechanics dealing with the computation of nonlinear problems. This is present in most general-purpose packages, such as ABAQUS, ADINA, ALGOR, DIANA … The main purpose is to develop efficient algorithms to carry out the computation of equilibrium paths. Many engineers view this approach as a heavenly solution to all problems, since all you need is brute-force computer capabilities.

Nonlinear algorithms are highly specialized and efficient for the computation of an equilibrium path. A good text in this field is Belytschko *et al.* [3] and the computational buckling approach is well presented by Bushnell [5].

There are two main tools incorporated into this class of predictive approaches: nonlinear analysis and eigenvalue analysis. The eigenvalue buckling problem starts with the computation of the pre-critical states u^F from the condition of equilibrium

$$\mathrm{K}\, \mathrm{u}^{\mathrm{F}} + p = 0 \tag{8}$$

Next, the stress field (σ^F) is evaluated. The results of this first stage are carried into a second stage to solve the eigenvalue problem

$$[\mathrm{K} + \lambda\, \mathrm{K}_{\mathrm{G}}\ (\sigma^{\mathrm{F}})]\ \phi = 0 \tag{9}$$

where K is the stiffness matrix, K_G the load-geometry matrix, Φ is the eigenvector (buckling mode) and λ^c is the eigenvalue (critical load). Basically, equation (9) can compute bifurcation from the linear fundamental path of equation (8).

Notice that this is a tool of analysis and not a phenomenological theory, so that no classification or explanation is attempted. It tells you how to compute, but it does not help in the understanding of the computed results.

3. PRACTICE: THE COMPUTATIONAL BUCKLING ANALYSIS OF THIN-WALLED STEEL TANKS

Steel tanks used to store oil and petrochemical products are very thin shells, simple in their overall geometry, and very costly. Perhaps the most significant behavior of a tank is its buckling strength: a tank may easily loose its geometry. There are many reasons to be concerned about such uninvited changes in the geometry: a floating roof may not slide along the side walls, a damaged geometry triggers imperfection-sensitivity, and finally there is a huge loss for each day a tank does not operate. The stakeholders are the owners of the tanks, environmental protection agencies, and the insurance companies.

Buckling of tanks is a field in which there is no chance of finding appropriate analytical closed-form solutions so that computational mechanics is a must. Tanks are designed by engineering firms with standard finite element computer packages and medium to low level engineering knowledge about the mechanics of the problem. The basic tools of analysis are linear/nonlinear static analysis, bifurcation-buckling analysis, and nonlinear dynamic analysis. The tools are available, thus we face here a problem of efficient use of the available software. In this section we review some examples of studies carried out with different theories in order to illustrate what phenomena may be identified with the models.

4. ELEMENTARY BUCKLING ANALYSIS FOR PRESSURE-INDUCED INSTABILITY

Vacuum pressure is a static load produced during the process of emptying a tank with insufficient venting. This form of buckling is usually associated to operational problems of a plant and is frequently found in practice. Koiter's predictive theory has been applied by Jorgensen [22] to evaluate isolated buckling loads and modes, and imperfection sensitivity under pressure. What features of real tanks are included in the analysis? A steel cylinder with $R=28m$, $L=24m$, and a wall thickness tapered between $h=0.032$ at the bottom and $h=0.012m$ at the top; the shallow cap itself is not part of the model (the finite difference discretization does not allow for a roof) and has been substituted by simply supported boundary conditions.

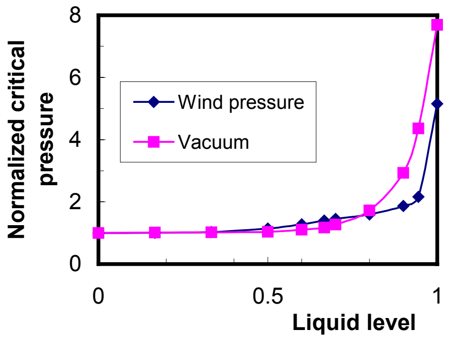

Figure 1. Influence of liquid level on the critical pressures of tanks. The results for wind pressure are for an open-top tank [10], while the results for vacuum [22] correspond to a simply-supported condition at the top.

In an independent study, Flores and Godoy [10] considered wind pressures on open-top tanks to address the influence of a liquid on the critical wind pressures, and the results are also plotted in Figure 1.

Results of the influence of the liquid on the critical pressure (normalized with respect to an empty tank) are shown in Figure 1 for the two loading conditions, and they show the same trend: if the tank is filled with liquid up to 50% of its height, the increase in critical pressure is of only 3 to 5%. This is of great importance both from the handling of tanks in practice and also for what studies are worth doing. The information in Figure 1 does not tell anything about other changes that may have occurred due to the liquid in the postbuckling behavior, or in the imperfections sensitivity. Such studies are reported by Jorgensen using Koiter's perturbation analysis [22].

5. STATIC OR DYNAMIC BUCKLING?

Wind acting on a tank produces a pressure variable both in time and space. Three-second wind gusts are considered as the basic information for design. The main questions regarding the computational modeling of wind-load buckling may be posed as: Is this a static or dynamic stability problem? How important is the influence of the roof? What is the influence of previous damage on the buckling strength of a tank?

The question about the importance of dynamic effects cannot be solved solely within the limitations of static theories, and we need to employ the criterion due to Budianski and Roth [4, 18] for dynamic buckling. Flores and Godoy [10, 11], have performed such studies previously for open-top tanks and silos.

For three-second gusts, an example of the nonlinear dynamic response of a tank under increasing values of step load is shown in Figures 2 to 6. The tank itself has a cylindrical part, a fixed roof and rafters supporting the roof, as indicated in Figure 2, and the finite element discretization with ABAQUS [17] employs about 12,600 shells elements. The space variation of pressures in the cylindrical part of the tank is constant in elevation and variable around the circumference, as in most works in this field [10], but for the pressures acting on the roof there is no clear indication as to what should be assumed. Here we consider the wind-tunnel pressures evaluated by MacDonald *et al*. [25], shown in Figure 3. The values of the pressures are scaled using the load parameter λ, which is the scalar that multiplies the pressure distribution shown in Figure 3. The maximum pressure in the reference case is 1KPa acting on the windward meridian on the cylindrical part of the shell. This pressure distribution is applied as a rectangular impulse (constant amplitude in time) with 3 sec duration. Notice that this problem is far more complex than the earlier investigations on simple cantilever cylinders.

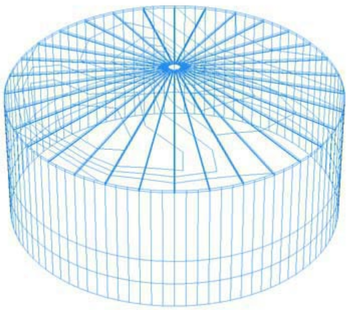

Figure 2. Model of the tank with fixed roof and rafter supports on the roof.

The nonlinear dynamic response has been computed using ABAQUS [17], and the radial displacement is shown in Figure 4 for a specific location in the cylindrical part (node 8938). According to the criterion for dynamic buckling, the response is computed for several values of the load parameter λ. For $\lambda = 2.50$ and $\lambda = 2.51$, the response shows small amplitude vibrations, of the order of the shell thickness. However, for $\lambda = 2.515$ we get a qualitative change in the response, with large amplitude vibrations starting at about t = 2.4 sec. This is an indication of a non-proportional change in the response due to a small increase in the load parameter. As the load parameter is increased, the shell exhibits instability, but the process starts earlier: for example, for λ = 2.52, large vibrations start at t = 1 sec, while for λ = 2.60 the time is reduced to approximately t = 0.5 sec. As the load parameter increases, the number of small-amplitude oscillations is reduced before the shell buckles.

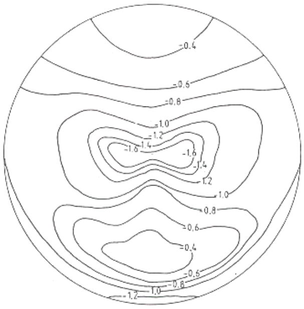

Figure 3. Pressure distribution assumed on the roof of the tank (after MacDonald *et al.*[25]).

Notice that Figure 4 has been plotted for a specific point of the shell, however, a similar behavior is obtained if other degrees-of-freedom (DOF) are considered, provided the nonlinear vibrations have a nonzero component in this DOF.

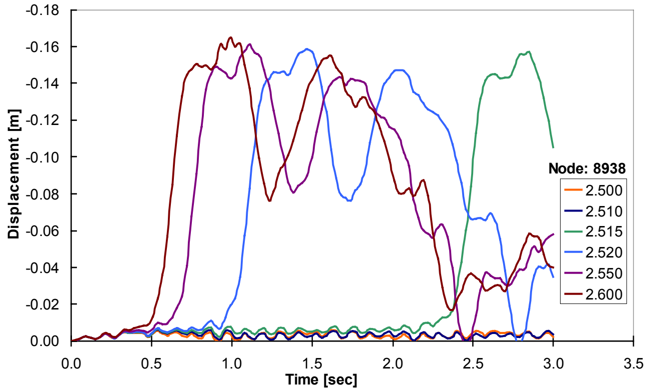

Figure 4. Nonlinear dynamic response for 3 sec wind gust assumed constant.
Each curve is for different pressure amplitude. Instability occurs for λ = 2.515.

The vibration pattern due to the nonlinear dynamic analysis, for λ = 2.515, at time t = 2.55 sec, is shown in Figure 5, while Figure 6 shows the vibrations for λ = 2.55, at time t = 3 sec. Figure 5 represents the onset of dynamic buckling, as this is the first stage for the lowest load for which instability occurs. Figure 6, on the other hand, is an advanced vibration pattern, and would only occur if the shell is suddenly slammed by the pressure λ = 2.55. The first figure shows a symmetric pattern of vibration, with blue deflections towards the inside and red towards the outside of the shell. The second figure illustrates a break in the symmetry, due to coupling of modes in an advanced postbuckling state.

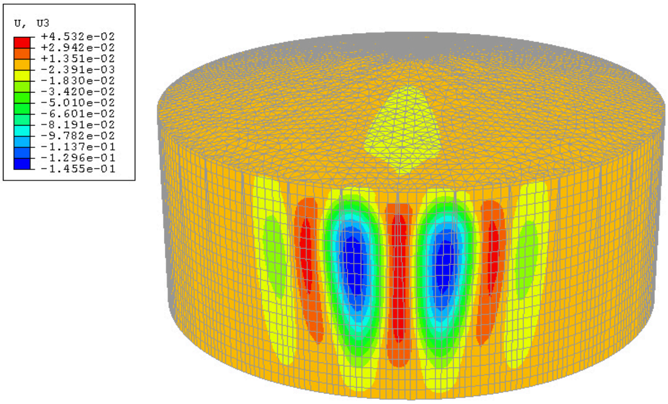

Figure 5. Nonlinear dynamic analysis, deflected pattern for λ = 2.515, at time t=2.55 sec.

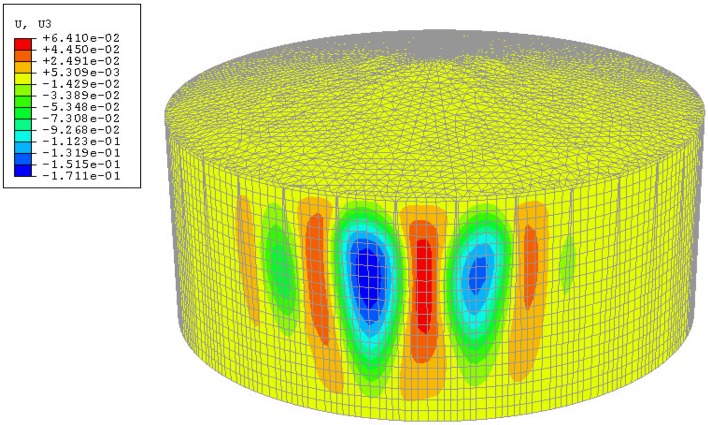

Figure 6. Nonlinear dynamic analysis, deflected pattern for λ = 2.55, at time t=3 sec.

Static models. The above procedure is time consuming and computationally expensive, since several cases of dynamic nonlinear response have to be solved. Thus, a simpler approach has been attempted, using a static, geometrically nonlinear analysis of the shell with the same pressure distribution but monotonically increasing in time.

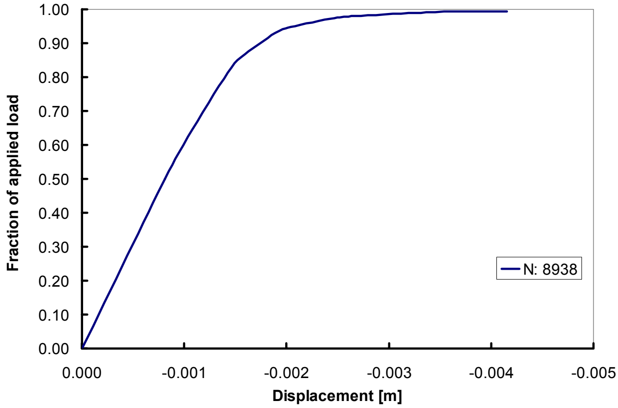

Figure 7. Static nonlinear analysis. The maximum load is λ = 2.55.

The equilibrium path for load parameter versus a DOF of the structure is shown in Figure 7. Here the same DOF considered in Figure 4 has been chosen. The path is nonlinear and reaches a maximum at λ = 2.55, that is a value slightly higher than the dynamic buckling pressure identified in the nonlinear dynamic study (λ = 2.515), the difference being of only 1.4%. According to Koiter's classification theory, this is an unstable symmetric bifurcation. A symmetric shape is found at the onset of instability, as illustrated in Figure 8. Figures 5 and 8 show similar patterns of behavior of the shell, so that not only the maximum load but also the buckling shape is well predicted by a nonlinear static analysis of this problem.

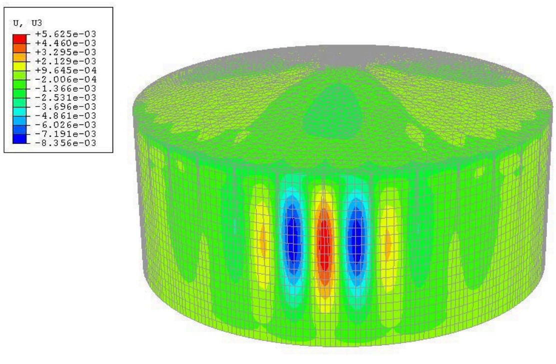

Figure 8. Static nonlinear analysis, deflected shape for λ = 2.55.

The reasons for the close agreement between nonlinear static and dynamic solutions are to be found in the natural frequencies of the shell [26]. For the present configuration, the period of vibration of the shell is 0.36 sec, thus far from the excitation considered of 3 sec.

Bifurcation buckling analysis has also been investigated for this problem using ABAQUS [17], and the results lead to a lowest critical load λ = 2.6107 (see Figure 9). A second mode (Figure 10) is identified at almost the same value λ = 2.6133. There is a small difference between static nonlinear and bifurcation buckling (2.3 %).

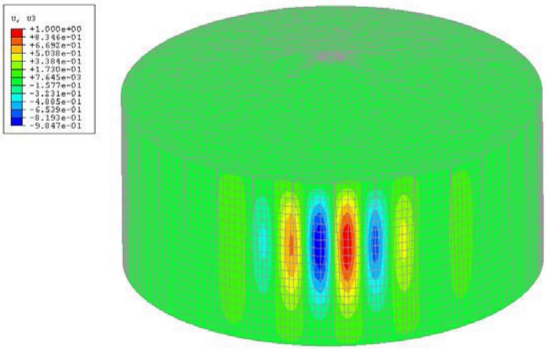

Figure 9. Bifurcation buckling, first mode, λ = 2.6107.

A lesson learned is that this is mainly a static problem. It is only now that one can confidently employ static predictions for this kind of environmental action. In the previous studies, Koiter's predictive approach has not been used, but a nonlinear computational mechanics approach has been adopted. However, Koiter's explanatory theory was present throughout the study: all explanations are possible thanks to the framework provided by the stability theory due to Koiter.

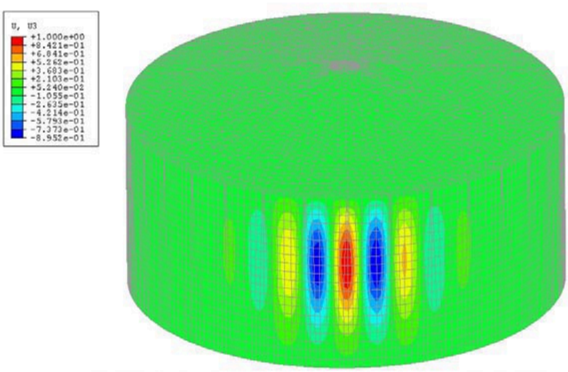

Figure 10. Bifurcation buckling, Second mode, λ = 2.6133.

6. INITIAL VERSUS ADVANCED POSTCRITICAL BEHAVIOR

The static response in the previous section was limited to the close vicinity of the critical state. This is reasonable, especially for the case of a tank with a roof, but in more flexible structures one may want to consider the behavior along the postcritical equilibrium path.

Let us consider an open-top cylindrical tank, again under wind load (Figure 11).

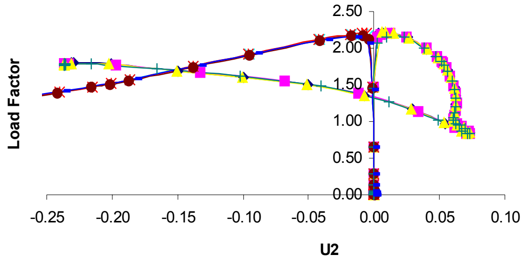

Figure 11. Nonlinear equilibrium paths for an open-top tank with initial load imperfections [20].
R=19m, Height=7.6m, h=0.01m.

The equilibrium paths for several load imperfections computed with ABAQUS are shown in Figure 11, and an unstable symmetric bifurcation behavior is identified, according to Koiter's classification theory. In the close vicinity of the bifurcation, a symmetric behavior is computed similar to what was observed in Figure 7; however, as the deflections increase in the postcritical path, a more elaborate behavior occurs: The left branch (negative displacements in the figure) tends to decrease, showing an unstable postcritical behavior. The right branch (positive displacements) decreases until a certain value, at which a snap occurs and the structure changes from positive to negative displacements. This is a feature that could not be predicted by Koiter's initial poscritical analysis, since it occurs far from the critical state and is due to advanced nonlinearity in the response.

An advanced postcritical behavior is also required in other buckling problems of steel tanks. Often, tanks are constructed near a shore with poor soil conditions and may face support settlements. Traditionally, this problem has been viewed in the literature as a linear static behavior, so that the geometric nonlinearity is not accounted for. But recent studies [15,16] have shown that the nonlinear behavior of the shell plays an important role in the response, with out-of-plane deflections of the order of five or more times the settlement in the vertical direction. A lesson learned is that bifurcation is highly relevant in this case and may provide good estimates of the buckling capacity.

7. IMPERFECTION SENSITIVITY

One of the crucial aspects of Koiter's theory was the inclusion of imperfection sensitivity into the field of buckling. A key feature in Koiter's explanatory theory is the sensitivity of buckling loads to the influence of small imperfections in some shells. In the previous case of a tank with a stiff roof, the shell under wind pressure is almost insensitive to imperfections. But for open-top tanks, the shell may display high sensitivity, as reported in a recent paper [14].

How this can permeate into practice for other problems can be illustrated with respect to the results of Figure 1. Let us plot again the results but now with a critical load normalized with respect to the tank full of liquid in the vertical axis, and the liquid level in the horizontal axis. What we now see is a diagram of sensitivity of the critical load to changes in the level of liquid inside the tank. The shape of the diagram is not surprising: it is a typical diagram with a singularity at the origin (1 in this case) and which is highly sensitive to small changes and almost insensitive to large changes, for which there is a plateau. It is possible to look for this form of diagrams only within the explanatory theory of Koiter.

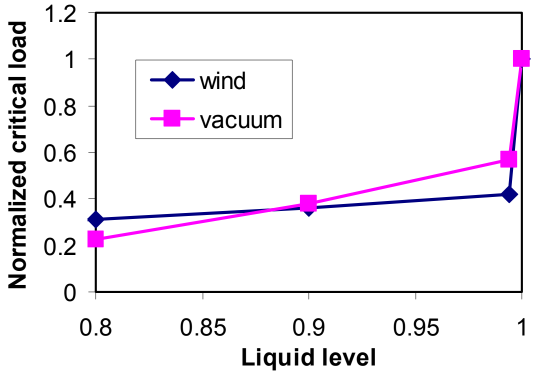

Figure 12. Sensitivity of critical load to a decrease in the liquid level filling the tank. Data as in Figure 1.

In practice this means that little contribution can be expected from the liquid to stabilize the tank, so that no real advantage can be gained by filling the tank with say half its contents.

8. POSSIBILITIES OF IMPLEMENTATION OF CROLL'S APPROACH

Croll's approach has not been mentioned in the sections about applications mainly because it is more a predictive tool than an explanatory theory. The use of Croll's theory in engineering practice is conditioned to the availability of a suitable predictive tool to carry out the analysis. In most cases, the reduced energy lower bound approach has been used with an analytical solution of the buckling modes, so that simple loading and boundary conditions can be taken into account. Furthermore, in cases of more complex structures, such as those shown in Figure 2 with compound shells and rafters on the roof, there are no analytical solutions available that could be used for the computation of the total potential energy. Thus, although this approach is physically sound and convenient, it faces the shortcoming of not being easy to use in practice. Only if the engineer is ready to develop his/her own computer program for the lower bound analysis, then it may be employed with advantage over a full nonlinear analysis.

An alternative approach would be to consider a homogeneous steel shell as if it was made with a laminated material. In a laminated material, modeled by the classical lamination theory developed for composite materials [21, 1], the stiffness of an individual lamina is computed as in plane stress behavior. The lamina is rotated to the global directions of the laminate. Individual contributions from laminae are assembled into a laminate and define a constitutive relation of the form

$$\begin{Bmatrix} N \\ M \end{Bmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{Bmatrix} \varepsilon \\ \chi \end{Bmatrix} \tag{10}$$

where {N, M} represent the membrane and bending stress resultants; {ε, χ} are the membrane strains and changes in curvature of the shell; and [A, B, D] contain the coefficients of the laminate. For a symmetric laminate, the bending-extension coupling vanishes (B = 0), and we are left with the uncoupled membrane and bending contributions. One of the advantages of this indirect definition of the material is that now one can perform separate evaluations of the individual terms of the membrane and bending contributions by adequately controlling the individual coefficients of the constitutive relations (10). Rather than deleting terms in the energy, one could delete coefficients in the constitutive relation.

Consider again the eigenvalue analysis (equations 8 and 9). The lower bound approach requires the computation of the fundamental equilibrium path (usually a linear path in equation 8) using the full energy contributions to the stiffness of the shell. But for the eigenvalue problem (equation 9), we need separate computation of K and K_G. Using a general-purpose finite element program, one needs to work in two steps, with different shell properties in each. ABAQUS, for example, allows the use of steps in an eigenvalue analysis, but does not easily allow for changes in the properties from one step to the next.

We have not developed these ideas in detail at present; however, it is our feeling that if the reduced energy approach is to be adopted in practice, ways to implement it into standard finite element codes should be found. This section outlines a possibility of using laminate elements to account for the energy contributions of basically homogeneous shell structures.

9. CONCLUSIONS

This paper stresses the importance of explanatory theories for the computational buckling analysis of shells. It is shown that the algorithms of nonlinear computational mechanics that are implemented in most general-purpose finite element programs are necessary but not sufficient to perform adequate instability studies, and that it is the power of explanation in Koiter's theory that allows to make sense of results.

As a predictive tool, several authors have implemented Koiter's perturbation analysis in conjunction with finite elements for the analysis of shells; however, its use has been limited to specific-purpose programs not available for commercial use.

Croll's reduced energy approach has suffered the same limitations in practice. However, some possibilities are indicated in this paper that may help to turn this into a practical tool: The use of classical lamination theory for the solution of shells made with homogeneous materials. This indirect approach may deserve closer attention to model the reduced-energy contributions in which some of the energy contributions are dropped.

The framework of explanatory theories and predictive theories is considered in practice for the analysis of thin-walled, steel tanks to store oil and petrochemical fluids. Simple parametric studies of bifurcation buckling may be enhanced by considering them with respect to the imperfection-sensitivity of shells. Thus, a case of influence of liquid inside a tank as a stabilizing factor can be seen as the imperfection-sensitivity of tanks full of liquid with respect to a decrease in the liquid level. This shows a high sensitivity, so that for practical purpose it may not be convenient to speculate with the liquid level as a positive factor in the event of a hurricane.

To understand the behavior of shells under wind gusts it is necessary to enlarge the theoretical frame and consider dynamic buckling. However, the results show that the problem behaves essentially as a static one, and inertia forces de not play an active role because the frequencies of excitation are far from the natural frequencies of the shells. In this case, Koiter's theory becomes a good theoretical framework for the analysis.

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REFERENCES

[1] E.J. Barbero, *Introduction to Composite Materials Design*, Taylor and Francis, Philadelphia, PA (1999).
[2] R.C. Batista and J.G.A. Croll, "A design approach for unstiffened cylindrical shells under external pressure", *Proc. Int. Conf. Thin-Walled Structures*, Ed. J. Rhodes, Elsevier (1979).
[3] T. Belytschko, K.W. Liu and B. Moran, *Nonlinear Finite Elements for Continua and Structures*, Wiley, New York (2000).
[4] B. Budianski, "Dynamic buckling of elastic structures: Criteria and estimates", in *Dynamic Stability of Structures*, Ed. G. Herrmann, Pergamon Press, pp. 83-106 (1967).
[5] D. Bushnell, *Computerized Buckling Analysis of Shells*, Martinus Nijhoff, Dordrecht (1985).
[6] J.G.A. Croll, "Shell buckling: A return to basic mechanics", in *Applied Mechanics in the Americas*, vol.1, American Academy of Mechanics, pp. 410-417 (1995).
[7] F.G. Flores and L.A. Godoy, "Instability of shells of revolution using ALREF: Studies for wind loaded shells", In: *Buckling of Shells in Land, in the Sea and in the Air*, Elsevier Applied Science, Oxford, pp. 213-222 (1991).
[8] F.G. Flores and L.A. Godoy, "Elastic postbuckling analysis via finite element and perturbation techniques, Part I: Formulation", *Int. Journal Numerical Methods in Engineering*, vol. 33, pp. 1775-1794 (1992).
[9] F.G. Flores and L.A. Godoy, "Elastic postbuckling analysis via finite element and perturbation techniques, Part II: Application to shells of revolution", *Int. Journal Numerical Methods in Engineering*, vol. 36, pp. 331-354 (1992).
[10] F.G. Flores and L.A. Godoy, "Buckling of short tanks due to hurricanes", *Journal Engineering Structures*, 20(8), pp. 752-760 (1998).
[11] F. G. Flores and L. A. Godoy, "Forced vibrations of silos leading to buckling", *Journal of Sound and Vibration*, 224 (3), pp. 431-454 (1999).
[12] L.A. Godoy, *Theory of Elastic Stability: Analysis and Sensitivity*, Taylor and Francis, Philadelphia, PA (2000).
[13] L.A. Godoy, "Sobre el sentido del progreso en la teoría de estabilidad elástica", *Mechanics*, vol. 31 (3-4), pp. 30-52 (2002).
[14] L.A. Godoy and F.G. Flores, "Imperfection sensitivity of wind loaded tanks", *Int. J. Structural Engineering and Mechanics*, vol. 13(5), pp. 533-542 (2002).
[15] L.A. Godoy and E.M. Sosa, "Deflections of thin-walled storage tanks with roof due to localized support settlements", In: *Advances in Structural Engineering and Mechanics*, Tecno Press, Seoul (2002).
[16] L.A. Godoy and E.M. Sosa, "Localized support settlement of thin-walled storage tanks", submitted for publication (2002).
[17] H.D. Hibbit, B.I. Karlson and Sorensen, *ABAQUS/ User's Manual*, Hibbitt, Karlsson and Sorensen, Inc. 1997.
[18] N.J. Hoff, "Dynamic stability of structures", in *Dynamic Stability of Structures*, Ed. G. Herrmann, Pergamon Press, pp. 7-41 (1967).
[19] K. Huseyin, *Vibrations and Stability of Multiple Parameter Systems*, Ivaordhoff, Alphen (1978).
[20] R. Jaca, L.A. Godoy and F.G. Flores, "Analisis postcritico de tanques de pared delgada bajo cargas de viento", *I South American Congress on Computational Mechanics*, Paraná, Argentina (2002).
[21] R.M. Jones, *Mechanics of Composite Materials*, Hemisphere Publishing, New York (1975).

[22] F. Jorgensen, “Buckling behavior of a liquid storage tank”, *Thin-Walled Structures*, vol. 1(4), pp. 309-323 (1983).
[23] I. Lakatos, *The Methodology of Scientific Research Programmes*, vol. I, Cambridge University Press, Cambridge, UK (1978).
[24] W.T. Koiter, *On the Stability of Elastic Equilibrium*, Ph. D. Thesis, Delft Institute of Technology, Delft, Holland (in Dutch) (1945). There is an English translation by NASA, 1967.
[25] P.A. MacDonald, K.C. Kwok, J.D. Holmes, “Wind loads on circular storage bins, silos and tanks: I. Point pressure measurements on isolated structures”, *Journal of Wind Engineering and Industrial Aerodynamics*, vol. 31, pp. 165-188 (1988).
[26] J.C. Virella, L.A. Godoy and L.E. Suárez, “Influence of the roof on the natural frequencies of steel tanks”, *Journal of Sound and Vibration*, submitted for publication (2002).
[27] S. Yamada and J.G.A. Croll, “Buckling and postbuckling characteristics of pressure-loaded cylinders”, *ASME Journal of Applied Mechanics*, vol. 60, pp. 290-299 (1993).