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BORE EXPANSION IN A CIRCULAR SHEET OF ELASTOPLASTIC MATERIAL USING SLOW ELASTOVISCOPLASTICITY THROUGH THE DIFFERENTIAL EQUATIONS ON A MANIFOLD METHOD

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Abstract. *In this work the differential equations on a manifold method (DEM) was used to determine different mechanical responses of elastic-viscoplastic materials for the particular case of a bore expansion in a circular plane sheet subjected to radial tensile uniform displacements at the outer edge. In particular, the large deformation in-plane strain and stresses were determined for a bore expansion ratio of 1.6, consideration is given to anisotropic effects, and hardening. The DEM strategy consists in giving approximate finite element representation to deformations, effective plastic deformation and to the displacement functions, at the same time the constitutive equations of large deformation hyper-elasto-viscoplasticity are approximated by collocation at the centroids of the triangular finite elements used. The result of these approximations is the generation of a system of algebraic-differential equations. In addition, since the formulation is based on the principle of virtual work, the fundamental requirement of equilibrium is guaranteed at all computational steps using the described procedure. The method, initially devised for viscoplastic materials, is extended herein for elastoplastic situations, where deformation rates are negligible. To this end very slow deformation rates were used, thus simulating the elastoplastic response, as a limiting case, using very slow elastoviscoplasticity. The results using the proposed strategy exhibit excellent agreement when compared with existing accepted results using different methods.*

1. INTRODUCTION

Using a known approach in the fields of plasticity and viscoplasticity, the result of discretization by displacement based finite elements (FE) of the equations of large deformation plasticity lead to the formation of differential algebraic systems of equations (DAE's), this results from the fact that discretization of the equilibrium equations results in non-linear algebraic systems of equations and discretization of the constitutive equations result in systems of differential equations. The differential equations on a manifold method (DEM) [1,2,3] also results in a DAE system, however, in this case the plastic strains and the effective plastic strains are given FE representation as well. Geometrically speaking the equilibrium equations generate a manifold in a multidimensional space on which the differential equations trace out paths. Of these paths, the one that passes through the multidimensional point representing the original state of the continuum is the particular solution to the DAE at hand. The above ideas are used herein to solve the radial stretching of a circular bore in a circular sheet of elsastoviscoplastic material. Due to symmetry only one fourth of the plate was used in the numerical experiments. Figure 1 shows the geometry and boundary conditions of our problem.

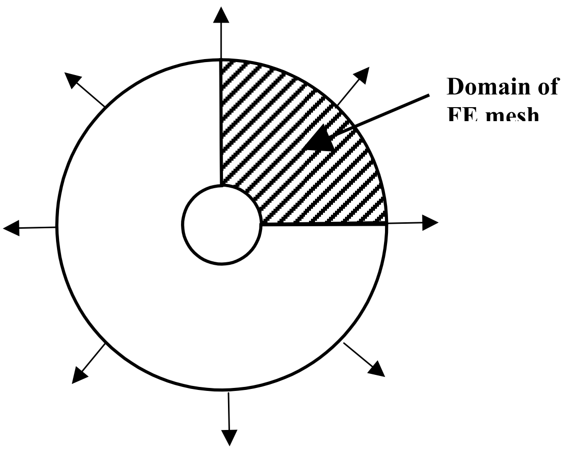

Figure 1. Circular uniform thickness sheet with circular bore subjected to radial pull.

For α, β, γ and δ assuming the values of 1 and 2, the displacement vector is given by

$$u(x_1, x_2, t) = u_\alpha(x_1, x_2, t) i_\alpha \tag{1}$$

(x_1,x_2) and i_α represent the fixed Cartesian plane and Cartesian base vectors respectively.

When the metal sheet deforms, the Cartesian system becomes curvilinear with the metric $g_{\alpha\beta}$.

The Lagrangian measure of strain is used in this work as follows,

$$E_{\alpha\beta} = \frac{1}{2}(g_{\alpha\beta} - \delta_{\alpha\beta}) = \frac{1}{2}(u_{\alpha,\beta} + u_{\beta,\alpha} + u_{\gamma,\beta} u_{\gamma,\beta}) \tag{2}$$

$\delta_{\alpha\beta}$ is the Kronecker delta.

Using the Kirchoff stress tensor $\tau^{\alpha\beta}$, the virtual work expression becomes

$$\int_A h_0 \tau^{\alpha\beta} \delta E_{\alpha\beta}\, dA = \int_s T^{\alpha} \delta\, u_\alpha\, ds \tag{3}$$

A, represents the original plate middle area, S, the initial boundary line, h_0, represents initial thickness and T^α, represents the components of the surface tractions. The summation decomposition

$$E_{\alpha\beta} = E^{(e)}_{\alpha\beta} + E^{(p)}_{\alpha\beta} \tag{4}$$

of elastic strains and plastic strains is used. In [4], for instance, the product of the strain deformation tensors is used instead. The elastic response is given by the approximate hyper elastic law

$$\tau^{\alpha\beta} = \frac{E}{1-\nu^2}[\nu\, g^{\alpha\beta} g^{\gamma\delta} + (1-\nu) g^{\alpha\gamma} g^{\beta\delta}] E^{(e)}_{\gamma\delta} \tag{5}$$

with R representing the anisotropy coefficient, the plastic response is specified according to the normality rule

$$\dot{E}^{(p)}_{\alpha\beta} = \frac{\dot{\bar{\varepsilon}}}{\tau}\left(\frac{1+2R}{1+R} g_{\alpha\gamma} g_{\beta\delta} - \frac{R}{1+R} g_{\alpha\beta} g_{\gamma\delta}\right)\tau^{\gamma\delta}\,. \tag{6}$$

The effective stress is given by

$$\tau^2 = \left(\frac{1+2R}{1+R} g_{\alpha\gamma} g_{\beta\delta} - \frac{R}{1+R} g_{\alpha\beta} g_{\gamma\delta}\right)\tau^{\alpha\beta}\tau^{\gamma\delta} \tag{7}$$

the dot represents material derivative. The time rate change of effective strain is experimentally determined and is solved for from

$$\tau = K\,[(\bar{\varepsilon} + \varepsilon_0)^n + \eta \ln\,(1 + \dot{\bar{\varepsilon}}/\gamma\,)] \tag{8}$$

The solution of the above equations for N elements and M vertices is obtained by giving a FE approximate representation to the displacements generating a non-linear system of algebraic equations

$$\mathbf{F}(\mathbf{Y}, t) \tag{9}$$

in the 2M+4N variables

$$\mathbf{Y} = (\mathbf{U_1}, \mathbf{U_2}, \mathbf{E_{11}^{(p)}}, \mathbf{E_{12}^{(p)}}, \mathbf{E_{22}^{(p)}}, \bar{\mathbf{E}}) \tag{10}$$

the $\mathbf{U}_\alpha$ represent nodal displacement, $\mathbf{E}_{\alpha\beta}^{(p)}$ the element centroid plastic strains, and $\bar{\mathbf{E}}$ the element effective plastic strain.

The differential equations are determined by approximating the viscoplastic response equations at the element centroids resulting in a matrix differential equation of the form

$$\mathbf{B}(\mathbf{Y}, \mathrm{t})\,\dot{\mathbf{Y}} = \mathbf{G}(\mathbf{Y}, \mathrm{t}) \tag{11}$$

F and **B** with **G** constitute the DAE.

As mentioned, the DAE as constructed may be geometrically interpreted as follows, a manifold is created (a surface in a multi dimensional space, generated by the approximation of the non-linear equilibrium equations) on which the differential equations representing the constitutive equations trace possible solution paths. Since the initial physical state is the null state, and hence a solution point, the solution path must pass through the origin in the multidimensional space. As a result the solution path traced by the resulting ordinary differential equations is located on the equilibrium manifold and consequently every solution corresponding to each incremental step satisfies the fundamental requirement of equilibrium.

The solution of the problem of a bore expansion in a circular sheet, Figure 1, was determined using the strategy outlined in this section.

2. APROXIMATION OF PLASTICITY BY SLOW VISCOPLASTICITY

The mathematical model given in the equations 1-11 above represents the behavior of a fully viscoplastic material. In the solution contained in this work the concept of approximating regular plasticity through very slow viscoplasticity is used and thus extending the applicability of the DEM method to this broad class of materials in the context of large deformation theory. From a practical computational standpoint this idea implies that the time rate of the effective plastic strain in Equations (6) and (8) is made sufficiently small but in no case equal to zero by applying sufficiently low speed displacements at the boundary of the domain. By proceeding in this fashion the computational structure of the DEM is retained without the necessity of making major changes and hence retaining the possibility of solving the resulting DAE system of equations with existing reliable advanced software for the type of problem treated herein.

The material response utilized in this work, that is, hyperelasticity together with viscoplasticity could be considered similar to that used in reference [4], where both hyper-elasticity and large deformation viscoplasticity are considered in the context of large deformations.

3. RESULTS

The problem of a bore expansion in a circular sheet was solved to verify the reliability of the proposed solution strategy. The sheet was subjected to a uniform radial expansion at its outer edge, resulting in a bore expansion ratio of 1.6. The circular sheet was 0.20 m in diameter and the bore was 0.010 m. The reference solution can be found in [5], where the problem was solved using an entirely different solution technique.

The hardening law material properties that appear in Eq. 8, for an Aluminum killed steel and chosen to match the properties used in reference [5] were as follows, K = 494.8 Mpa, $\varepsilon_0 = 0.0$, R = 1.5, n = 0.222, γ= 0.000137/s, η = 0.0018, E = 206843.0 MPa. The boundary conditions are as follows the inner boundary is entirely free , the outer boundary was subjected to an outward radial displacement rate of 1.0 mm/s.

The TRIMESH [6] generated FE mesh is shown in Figure 2., where only one quarter of the plate is indicated as a result of the problem symmetry. This automatic mesh generator produces highly desirable near equilateral triangles. DASSL [7] was used as the solver for the DAE resulting stiff system of equations. For this particular case given that there are 119 nodes and 194 elements the resulting DAE contains 1014 equations based on the following equation count, 238 displacements at 2 displacement per node, 582, strain components at three components per element, 194, effective plastic strains at 1 per element, 194.

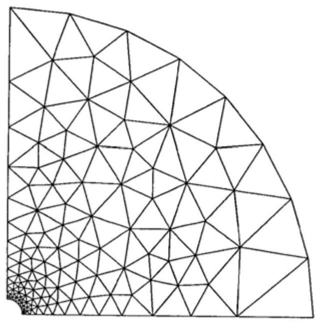

Figure 2. Finite element mesh used in the numerical solution.

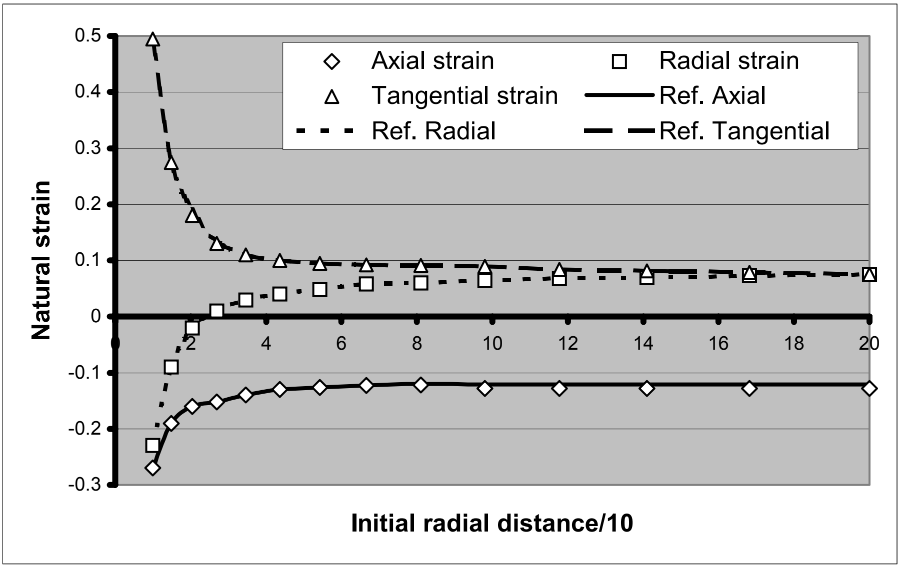

Figure 3. Comparison of present results with existing ones.

Figure 3 shows a comparison of the results using the present strategy with published values given in [5], labeled as reference values. As it can be verified in this Figure there is an excellent results agreement when using these entirely different solution schemes.

4. CONCLUSIONS

The solution of a benchmark type of problem is presented, namely, the radial plastic stretching of a circular disk with a circular centered bore for large deformation theory viewed as an approximate limiting case of a viscoplasticity problem, when using very slow strain rates in the context of a method afforded by the DEM solution strategy.

The results obtained using the stated approach compare rather well with existing results used as a reference, hence verifying that using slow viscoplasticity, in the sense described herein, is a valid method to approach the solution of plasticity problems.

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