SIMULATION OF THE DRYING STRESSES IN WOOD

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Abstract. The drying of solid wood and associated stresses are simulated by applying the Control Volume Finite Element Method (CVFEM) in a transversal section of solid wood on the radial/tangential plane. The transport of moisture content and stresses produced by its gradients associated with the phenomena of shrinkage and mechanical sorption were modeled simultaneously. In particular, we used a CVFEM program (Fortran 90) that allows integrating a differential equation of non-linear transient diffusion, defining triangular finite elements with linear interpolation of the independent variable within itself. The model is validated by comparing the experimental and analytical results. Finally, we show the original results of the simulation applied to the drying of solid wood.
1 INTRODUCTION

The present study focuses on heat and mass transfer coupled with strain/stress problem during drying process in terms of modeling and simulating the drying of solid wood. A collection of related works are given by Turner and Mujumdar (1997) and an updated review of these methods are given in Hernandez and Quinto (2005).

In the present work the potential model Cloutier and Fortin (1994) was adopted to simulate the transport of moisture content within the wood as described by author Salinas et al. (2004).

The effects of heat and mass transport cause strain/stress within the wood. Modeling this phenomenon is a complex process due to the effects that the drying process produces on the wood. This leads to stresses that cause permanent and transitory deformations due to variations of moisture contents.

The models proposed for wood focus mainly on deformations caused by the transport of energy (temperature) and mass (moisture content). Some works (Perre et al., 1993; Chen et al., 1997; Pang, 2000 y 2007) propose one-dimensional models for determining the deformations resulting from heat and mass transport; notably, deformation by shrinkage and mechanical sorption. Likewise, in two-dimensional (Turner and Ferguson, 1995; Lin and Cloutier, 1996; Ferguson, 1998; Kang and Lee, 2004) and three-dimensional models have been proposed for deformation Ormarsson et al. (2003).

Numerically, we use the Control Volume Finite Element Method (CVFEM) to solve the transport and deformation equations induced during the drying process. In general, the method CVFEM consists of a Finite Volume that is made up with Finite Elements. This model offers advantages related mainly to its intrinsic quality of conservatively given by Finite Elements Method and the topological versatility bestowed by Finite Elements Method Baliga B.R. and S.V. Patankar (1983).

Thus, we consider linear orthotropic variations of the properties and independent variables within the Finite Element, considering the discrete variable centered on the Control Volume. The numerical approach leads to the formulation of linear algebraic equations systems that are solved through iterative and direct methods (Gauss Saidel with SOR and Gauss Elimination).

The aim of the present work is concerned with simulation of the drying/stress problem, following systematic variations of geometric and physical parameters for the analysis of stability and consistency of the algorithms developed. Moreover, we validate the results obtained by comparing them with the experimental, numerical and analytical data available in the literature.

2 PHYSICAL MODEL

We study a physical model of the wood strain/stress problem during isothermal drying process. We consider the non-uniform transitory effects induced by the variation in the moisture content (M); that is: stress (σij), strain (εij), and displacements (ui = (u,v)).

As shown in Figure 1, we consider a transversal two-dimensional section of wood on the radial-tangential plane. The properties are given in Table 1 (Cloutier et al., 1992). The dimensions of this piece of wood are: wide L=0.045 (m) and thickness H=0.045 (m).

The initial and contour conditions are: a) for the problem of moisture content transport, initial moisture content of M=Mini, Neumann-type contour conditions of no-flow on the symmetry axes (x=L/2 and y=0), and surface convection of x=0 and y=H/2; and b) for the strain/stress problem, a non-deformed initial state (σij=εij=ui=0), Dirichlet-type contour conditions on the symmetry axes (ui=0 on x=L/2 and v=0 on y=0), and free contour conditions on the surfaces x=0 and y=H/2.
Figure 1: Diagram of the physical problem

Transversal section of wood on the radial-tangential plane with non-uniform transient variations in moisture content.

<table>
<thead>
<tr>
<th>Drying Temperature (°C)</th>
<th>20</th>
<th>35</th>
<th>50</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convection Coef. (kg²/m²·s·J)</td>
<td>$4.43\cdot10^{-10}$</td>
<td>$5.82\cdot10^{-10}$</td>
<td>$9.36\cdot10^{-10}$</td>
</tr>
<tr>
<td>Specific gravity</td>
<td>0.407</td>
<td>0.427</td>
<td>0.419</td>
</tr>
<tr>
<td>Elasticity module (MPa)</td>
<td>250</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Shrinkage Coef. (1/°C)</td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Creep Coef. (1/Pa)</td>
<td>$-1.0\cdot10^{-7}$</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Poisson ratio (1)</td>
<td>0.25</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial moisture(%)</td>
<td>135</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Equilibrium moisture (%)</td>
<td>10</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: Wood properties

3 THE MATHEMATICAL MODEL AND NUMERICAL MODEL

The mathematical model for local variation of moisture content based on water potential concept Cloutier and Fortin (1994) was used. In particular, as implemented in Salinas et al (2004), the transport is described indirectly through the water potential $\psi$ (J/kg) considering wood to be a two-dimensional orthotropic medium on the plane $xy$. This can be expressed as:

$$\partial_t (c_r \psi) + \frac{\partial}{\partial x} \left( k_{xx} \frac{\partial \psi}{\partial x} \right) + \frac{\partial}{\partial y} \left( k_{yy} \frac{\partial \psi}{\partial y} \right) = 0$$

(1)

where $k_{xx}$, $k_{yy}$ conductivity ($\text{kg}^2\text{water} / \text{m}^3\text{wood-moist} \cdot \text{s} \cdot \text{J}$) and $c_r \equiv \frac{G_r \rho_r}{100} \frac{\partial M}{\partial \psi}$ capacity ($\text{kg}^2\text{water} / \text{J} \cdot \text{m}^3\text{wood-moist}$).

The physical transport parameters to be determined experimentally are conductivity $k_{xx}$ and $k_{yy}$ in the main directions $X$ and $Y$, respectively, and the variation of moisture content in relation to the potential ($\partial M/\partial \psi$).

1 Because the coefficients $c_r$, $k_{xx}$ and $k_{yy}$ are now dependent on the variable transported and, in the case of conductivity, also on direction, these configure non-linear, orthotropic mass transport.
For two-dimensional stress, induced by non-uniform distribution of moisture content, based on mechanical equilibrium Zienkiewicz and Taylor (2000) was modeled through an implicit function of five parameters that define an initial deformation $\varepsilon^0$: shrinkage $\alpha$, mechano sorptive creep coefficient $m$, stress $\sigma$, variation of concentration $\Delta C$ and, in the case of plane deformation, the Poisson ratio $\nu$. That is:

$$\sigma = D(\varepsilon - \varepsilon^0) + \sigma_0$$

Where

$$\varepsilon^0 = \beta \Delta C = \begin{cases} 
(\alpha + m \sigma_{xx}) \\
(\alpha + m \sigma_{yy}) \\
0 
\end{cases} \Delta C \text{ stress plane}$$

$$\varepsilon^0 = \beta \Delta C = \begin{cases} 
(1 + \nu) (\alpha + m \sigma_{xx}) \\
(1 + \nu) (\alpha + m \sigma_{yy}) \\
0 
\end{cases} \Delta C \text{ strain plane}$$

The above mathematical models were integrated numerically using Control Volume Finite Element Method (CVFEM) to solve the transport and deformation equations induced during the drying process [14].

4 RESULTS

The results from the three simulations (moisture content transport in aspen wood, heat diffusion and thermal stresses in a steel plate, and moisture content transport and drying stress in aspen wood) are presented herein. The first two simulations are done in order to validate the computer codes for modeling, respectively, the transport of moisture content and stresses. The third simulation shows an application of this in the context of the wood drying process, which motivated this study.

4.1 Transport of moisture content in wood

Below, results of transient moisture content transport on a two-dimensional section of the aspen wood on the radial-tangential plane are shown. The drying problem is modeling as discussed in the item 2, it is illustrated in Figure 1 and its properties are given in Table 1.

Figure 2 shows moisture content simulated for $T$ equal to 35 (°C). We can see in Figure 2a the advance of the drying front: greater concentration of moisture content isolines reflect greater gradients. The orthotropic behavior can be observed in the lack of polar symmetry. The drying curves obtained by the present model with those experimental results [15] was shown by Figure 2a. The good agreement observed shows that this captures the essence of the physical phenomenon under study.
4.2 Heat diffusion and thermal stresses

The transient two-dimensional simulation of heat transport and the associated stresses was done for a steel plate (See Figure 2a). For heat transfer, the transport Eq. (1) might be written as:

\[ \frac{\partial T}{\partial t} = \alpha T \nabla^2 T \]  

Where, \( T \) is the temperature and \( \alpha \) the thermal diffusion coefficient.

The plate dimensions are: length \( L=0.6 \) (m) and height \( H=0.2 \) (m). The contour condition effects considered are: restricted displacement in the Y direction of layer AD (\( v=0 \) in \( y=0 \)), restricting in the X direction in the layer BC (\( u=0 \) in \( x=L \)), and freedom in the layers CD and DA. Naturally, point B is restricted in X and Y ((\( u,v=(0,0) \) in B). This problem has an analytical solution Boley and Weiner (1960).

Figure 3: 2-D thermal stress: a) Scheme of problem, b) Field of normal stress y c) Normal stress at BC layer.
The non-uniform distribution of temperatures produces stresses induced by dilatation, as it does when sustained over time (creep). The modeling of these stresses can be done in a equivalent manner to that of stresses presented for the variation of moisture content in wood. In particular, for an initial deformation $\varepsilon^0$ in function of: thermal dilatation $\alpha$, creep coefficient $m$, stress $\sigma$, temperature variation $\Delta T$, and for plane strain, the Poisson ratio $\nu$.

Figure 3 shows results for $t=1$ (h). It reveals the consistency in terms of the imposition of contour conditions and qualitative perform (Figure 3b). The analysis of convergence and consistency related to the size contrasted with the analytical solution, as shown in Figure 3c.

### 4.3 Moisture content and drying stresses

Below results are shown of computer code for moisture content coupled with strain/stresses problem in a piece of aspen wood during the drying process. For this, we determine, in each time of integration, the distribution of moisture content that allows a later calculation of the free deformations ($\varepsilon^0$) that motivate drying stresses.

Figure 4 shows spatial distributions of the calculation parameters for the strain/stress problem for the permanent state (400 h) and drying temperature equal to 50ºC. In Figure 4a we can appreciate the implementation of the contour conditions and as the greatest...
displacements concentrate on the restriction of a degree of freedom. Figure 4b shows in detail how the normal stresses concentrated in A, B, and C (see Figure 1), focusing on tension in A and C (surface) and compression in B (center). Figure 4c y d shows the transitory evolutions of the stresses in the center and the surface of the domain of calculation, points B and C of Figure 1. There, the dynamic of the variation of the intensities of the normal stresses can be appreciated. Basically, at the onset of drying on the surface and in the center, we observe a marked tension and slight compression, respectively. This contrasts with the final state of stresses at the end of the drying, in which the normal stresses are inverted. The latter is one of the relevant characteristics of the simulated physical problem. Of course, at initial drying the surface stress has one oscillatory performance because the critical drying condition.

5 CONCLUSIONS

The numerical results of two-dimensional simulation of moisture content transport, modeled according to water potential method, showed an effective modeling of transport phenomena for isothermal drying process at the three drying temperatures studied. The average differences between experimental and numerical data were less than 2%.

Furthermore, effective simulation was also done for the stress/strain phenomenon caused by free deformations and the associated creep phenomenon. In this case, it was used thermal stress problem, which analytical solution, to test the quality of algorithm. The agreement between analytical and numerical solution was very good.

Finally, the numerical results of the application (drying stress) showed, in qualitative term, a consistent simulation of the phenomenon studied: Moisture content transport correlated with the stresses associated with the moisture content gradients during the transient process of wood drying. In particular, large differences were not reported in residual stresses for the three drying temperatures studied. Nevertheless, there are hard differences in how the transitory stresses are developed, especially when stresses were compared in the center versus surface.

REFERENCES


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