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MODELING OF BEAM PLASTIC HINGE MECHANISMS IN CORRELATION WITH ANALYTICAL AND EXPERIMENTAL RESULTS

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Abstract. Plastic hinge in beams is a mechanism framed within the structural collapse of complex 3D lattice structures: car tubular chassis, buses, cranes, etc.; in this mechanism, the plasticity phenomenon (material nonlinearity) is mainly coupled with the local buckling phenomenon (structural nonlinearity). In this work, it is analyzed the extent to which the combination of these nonlinearities in finite element models is representative of the curves studied analytically and experimentally in the article "Bending collapse of rectangular and square Section Tubes" (Kecman, 1982) and Torsion (Trahair, 1997). The coupling is analyzed. The developed numerical models cover shell elements and beam element formulations, both implicit and explicit time integration schemes. The results show a high degree of correlation between the numerical models and the analytical-experimental results, which drastically reduces the need for physical testing. The work also extends the analysis to other types of nonsymmetric general sections, which are specific to the tubular structure of an electric vehicle. These tests show the limits of hybrid model technology (several types of elements combined), since it requires a load-discharge curve with hysteresis and does not allow for negative slopes, which are characteristic of the previously mentioned physical tests.

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1. INTRODUCTION

In this work, the problem of solving a complex frame structure collapse subjected to dynamic forces will be addressed, and the results compared between different finite element formulations. Beyond the minimum requirements that the structure must withstand for certification, the structure collapses in different ways that are of interest, with the aid of modelisation, this way contributing to an increasing passenger safety.

Many strategies have been developed for this type of modelling, including shell models, hybrid models (beams and nonlinear springs), and explicit models.

The objective of this work is to determine which beam strategy is more suitable for reproducing the shell elements collapse, along with possible physical experiments.

1.1. Geometric nonlinearity on beams

As a theoretical framework, the beam models will be solved by line elements with nonlinear geometry. Also, the beam formulation is based on Reissner's beam theory, which is theory of curvilinear beam elements as different from co-rotational formulation, allowing second-order effects like plasticity, buckling, etc. The theory deals with Intermediate Class of Deformations (Ibrahimbegovic, Vol. 122. 1995)

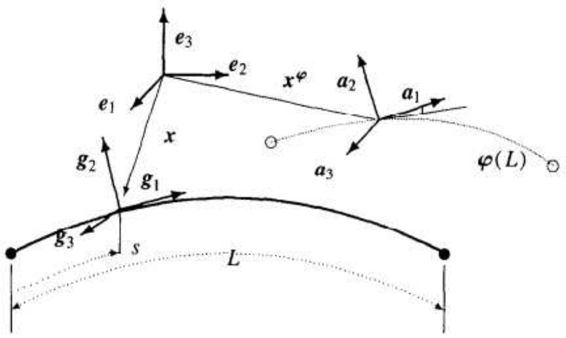

Figure 1. Three-dimensional curved beam.

In order to derive the equilibrium equations for the line element in Figure 1, intermediate class of deformations are used.

$$\bar{\varepsilon} = \bar{u}' + \frac{1}{2}(\bar{w}')^2 \tag{1}$$

It is assumed that the linear strains Ɛ are small, and also the angles of rotation are small, but not infinitesimally small (finite rotations). This way, allowing nonlinear phenomena to be modeled.

2. BENDING COLLAPSE OF BEAMS

The bending collapse mechanism in a beam is a process that begins with a local buckling after reaching a limit point $\sigma_{crit,}$, generally at the compression flange of the beam, that might be coincident with plasticity or not. This process varies depending on the type of section (open or closed), and it often occurs at the vicinity of a welded connection with other beam, or any kind of discontinuity.

The thumb rule for the thin-walled section of beams and shell is:

$$\frac{R_i(Radius_{inside})}{t\ (wall\ thickess)} \geq 10 \text{ (means thin walled p.v., pipe, etc.)} \tag{2}$$

The most cited and referenced work on plastic hinge mechanism is based on energy or variational method, and established a mechanism of square straight beams. On this work, (Kecman, 1983) the theoretical prediction of collapse limit of a beam section is enabled, and is developed for rectangular and square section tubes. Prior to this work, the critical load Pcrit at which the plastic hinge was determined experimentally. The case of thin-walled rectangular columns, based on Kecman's model, is treated on (Kim T.H., 2001) and it uses a kinematic method based on perfect plasticity. In the case or circular tubes, this work was performed by (Liu Y., 2008). The three-point bending collapse based on finite element models was performed in (Huang Z., 2018). The modelisation of plastic hinge and the formulation of superbeam elements was developed in (T. Wierzbicki, 1994). Kecman's work enables prediction of the hinge properties. A detailed analysis of the hinge mechanism and the deformed pattern, as well as correlation with experimental results, is developed on the author's work. The collapse mechanism is divided into three phases: the first phase being linear until the σcrit stress is reached, that can be different from yield stress. At this point, the section buckles, and the second phase (plastic hinge) begins, until reaching the third phase, called jamming.

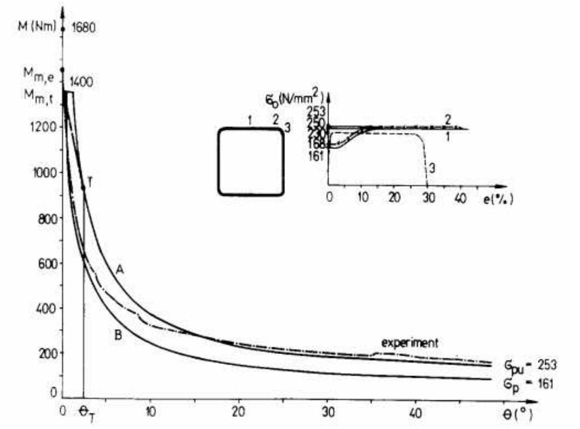

Figure 2. Theoretical (solid) and experimental (chain-dotted) curves of M vs. θ.

The basic parameters to build the curve in Figure 2 are the Maximum torque Mt, the maximum nominal flow stress σpu, and the angle of transition θt. With this curve, the hybrid finite element beam and nonlinear spring model can be built, and material properties defined.

3. TORSION AND BUCKLING OF BEAMS

In the plastic collapse analysis of torsion developed in (Trahair, 1997) the statement of the torsion problem for this work, is defined in a complete manner.

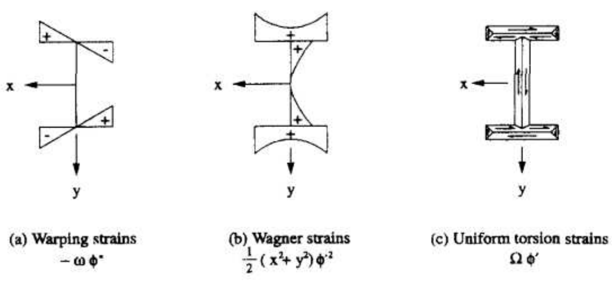

Figure 3. Strain variations around cross-section.

In Figure 3, linear elastic analysis of torsion is defined in terms of uniform torsion and warping, and Wagner strains appear when nonlinear inelastic strains develop in the torsion.

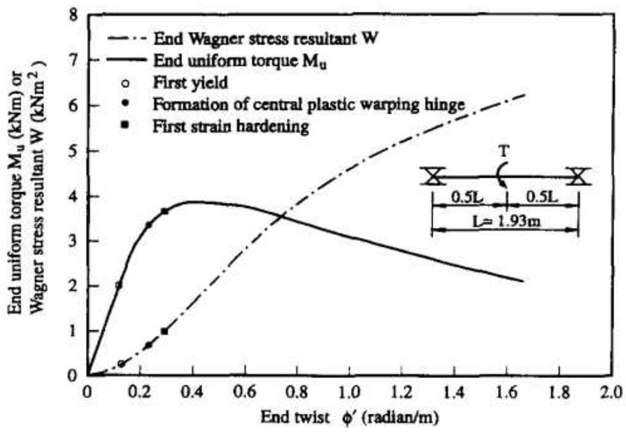

Figure 4. Effect of large twist rotations on Wagner stress resultant and uniform torque.

In Figure 4 we can see that when the collapse point is reached, the Wagner strains continue to increase while M is decreasing, in a phenomenon that shows similarity with bending. In this case, the warping torsion becomes non-uniform, developing a plastic hinge.

4. FINITE ELEMENT MODELLING STRATEGIES TO THE BUCKLING AND COLLAPSE MECHANISMS

The two strategies followed in this work are:

- Hybrid models consisting of beams and nonlinear spring elements (Reference: (Miles, 1976) and (McIvor, 1977)
- Beam elements with Explicit time integration (AUTODYN Software)

There are also other approaches like the superfolding element (T. Wierzbicki, 1994) also taken into account in this work.

5. CORRELATION WITH SHELL ELEMENTS-BENDING

Starting from Kecman's curves in bending and developing a shell (SHELL181 model with Ansys Mechanical Software), the simple beam test had been run with the same characteristics as the Kecman's tests. This model is used as correlation with a physical test in this work. The solvers used were Ansys Mechanical and Autodyn Explicit.

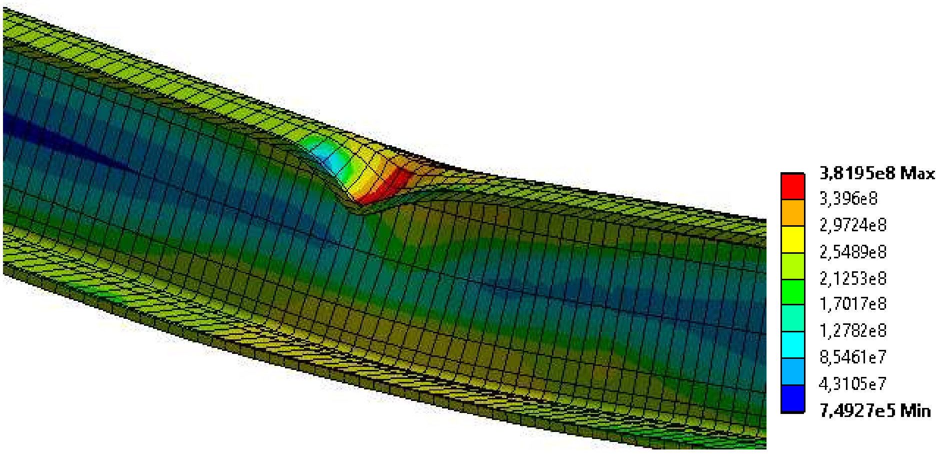

Figure 5. Equiv. Stresses [Pa] in a Static Structural model with plastic hinge formed.

The Static Structural result shown in Figure 5 shows a good correlation with the experimental results shown in Kecman's work. The shell models with SHELL181 formulation are capable of capturing the buckling of the section, and the deformation shapes described are coincident with the theoretical and experimental results.

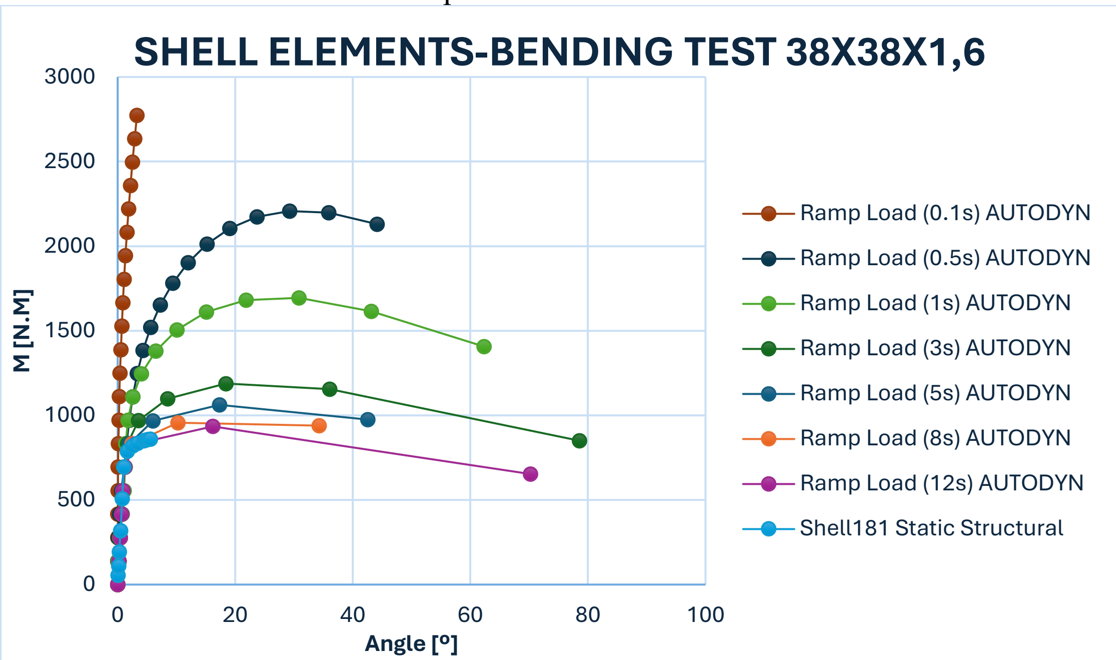

Figure 6. Moment vs. Angle for finite element models.

The plot in Figure 6 shows the strain rate effect from Static Structural, with the same M applied in a shorter time, from 12s. decreasing to 0.1s. The latter shows a different collapse mechanism, and the intermediate showing the same deformation pattern, but with the effect of inertial loads clearly affecting the curve.

6. CORRELATION WITH SHELL ELEMENTS-TORSION

A correlation between torsion (Trahair, 1997) and SHELL181 elements was also done and shown in this section. As in the Bending case, the model was run both with implicit and explicit approaches.

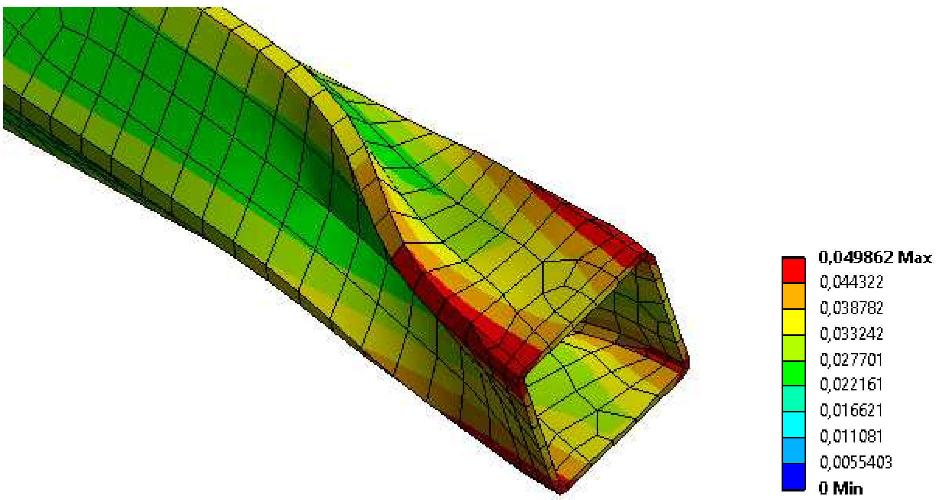

Figure 7. Torsion test deformations with SHELL181 elements. Units in [m].

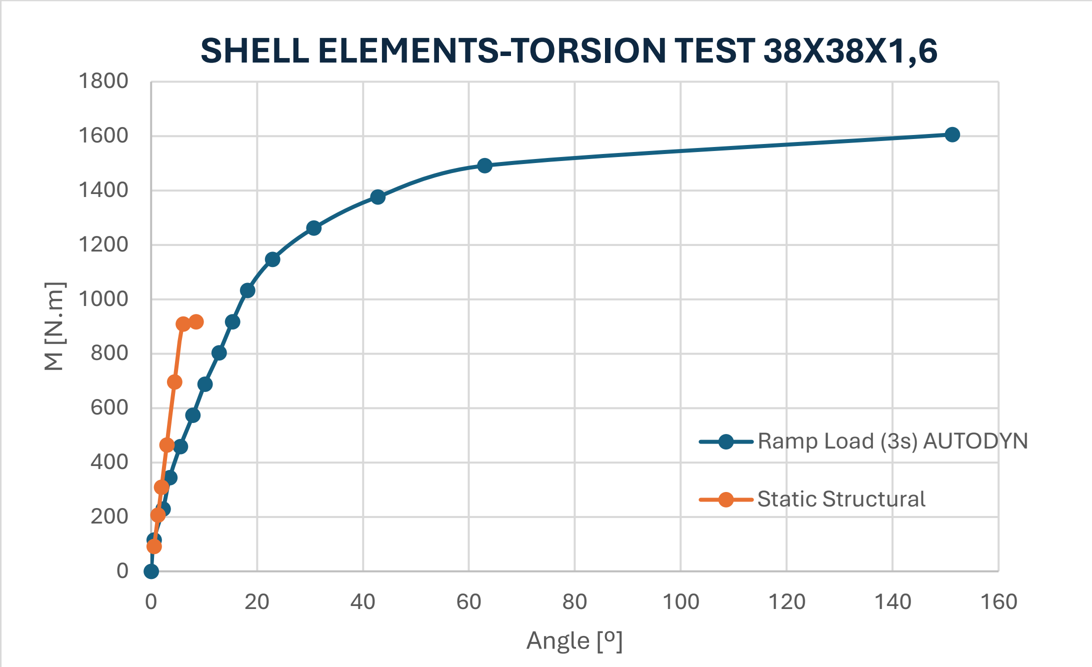

Figure 8. Torsion test. Comparison between implicit and explicit time integration results.

The torsion test shown in Figure 7 and Figure 8 are showing a particular shape, typical of the summation of all kinds of torsion: Coulomb, Saint-Venant, plastic Wagner strains, and due to de collapse, non-uniform torsion.

7. CASE STUDY: GENERAL SECTION

When analyzing a general section, there have been coupling between bending mechanisms and torsion mechanisms seen in the previous sections.

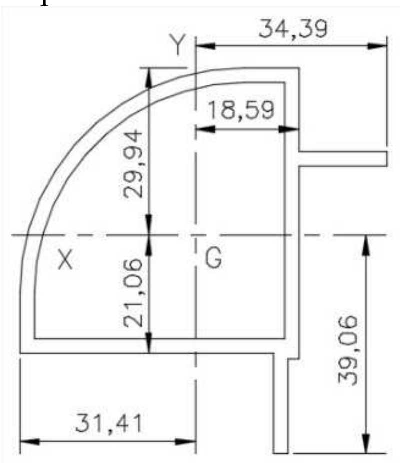

Figure 9. Section profile for the framed electric car structure. Dimensions in [mm].

In Figure 9 we can observe that the section has an average wall thickness of 2.5 mm, and an approximate length/t of 20. The section is closed but it has also open sectors.

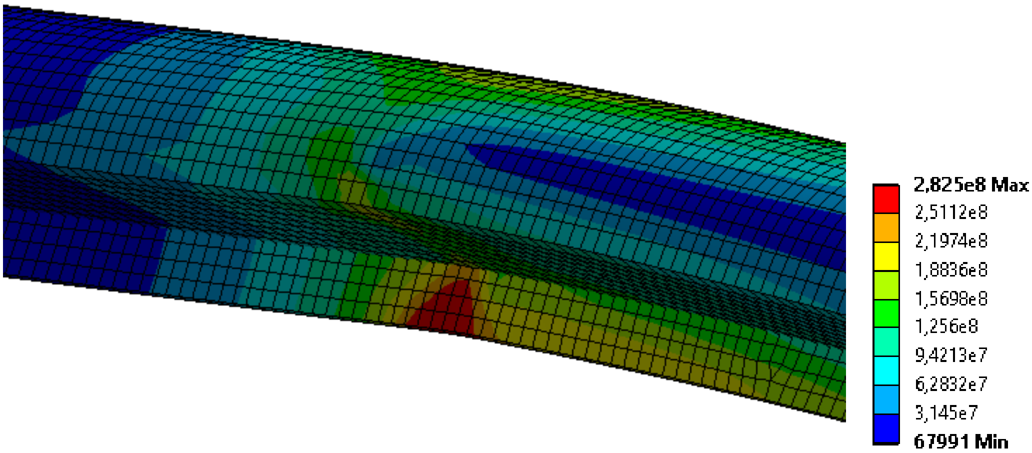

Figure 10. Equivalent Stresses in Bending Test on a general Section. Units in [Pa].

In Figure 10 we can see that the section buckles in a nonuniform trajectory, first buckling an open section, and the coupling between bending and torsion is very high, to the point that it is not clear which mode is causing primarily buckling.

8. CASE STUDY: FRAMED STRUCTURE (ELECTRIC CAR BODY)

The case study used in this work is a framed structure with the general section analyzed in the previous paragraph. It is the frame of a 2-seat electric city car. For this structure, several finite element models were built in order to analyze the beam finite elements response when compared to a reference model (in this case, shell finite elements). Due to the increasing complexity of this practical case study, the finite element method can be applied in different ways to capture the phenomenon of collapse sought for beam models.

The following types of beam models have been used:

- Fully beams model
- Hybrid model (beams combined with torsional springs (Miles, 1976) and (McIvor, 1977)
- Beam Resistance Model

Also, as a reference model:

- Pure Shell Elements model

8.1. Shell elements model

The shell elements models have been correlated with bending and torsion physical experiments, showing high precision modelling both stresses and plastic strains, and also capturing the buckled shape in accordance with rectangular and circular sections.

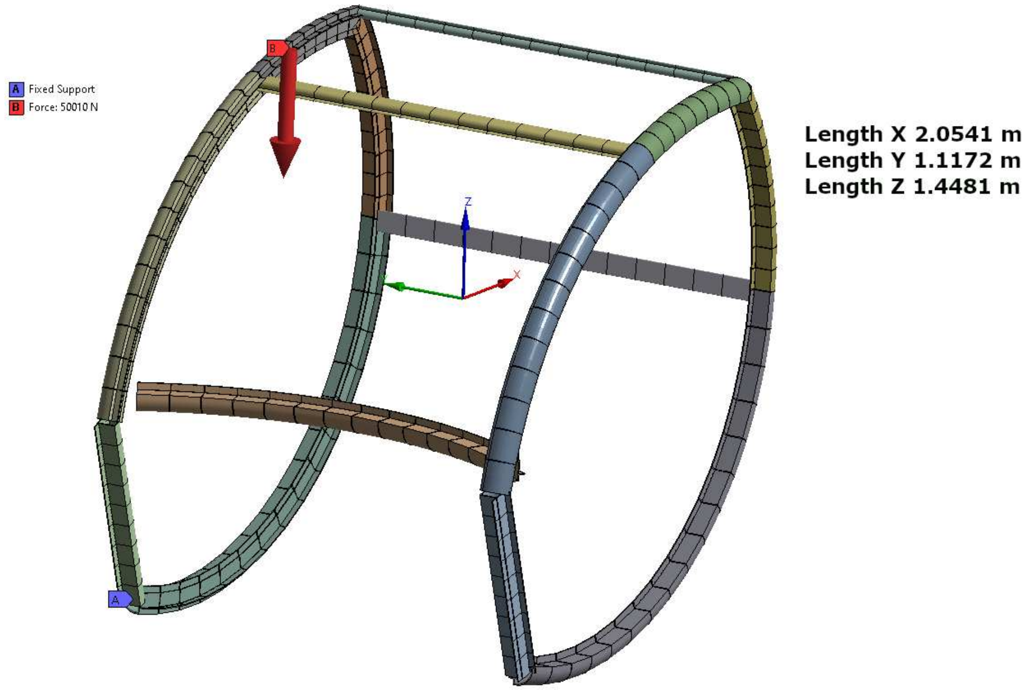

Figure 11. Problem Statement for roof collapse in a 2-seat electric car body tubular structure.

In Figure 11, we can see the mesh and boundary conditions for the chassis of an electric car model, built entirely with Aluminum Alloy NL material model.

The problem was run with Ansys Autodyn explicit Solver, with bonded Contact between the different parts of the chassis frame. The load pattern was linear application of force from 0 to maximum in 1 s, and then unloading in 1 s. returning linearly to 0, with a total simulation time of 2 s.

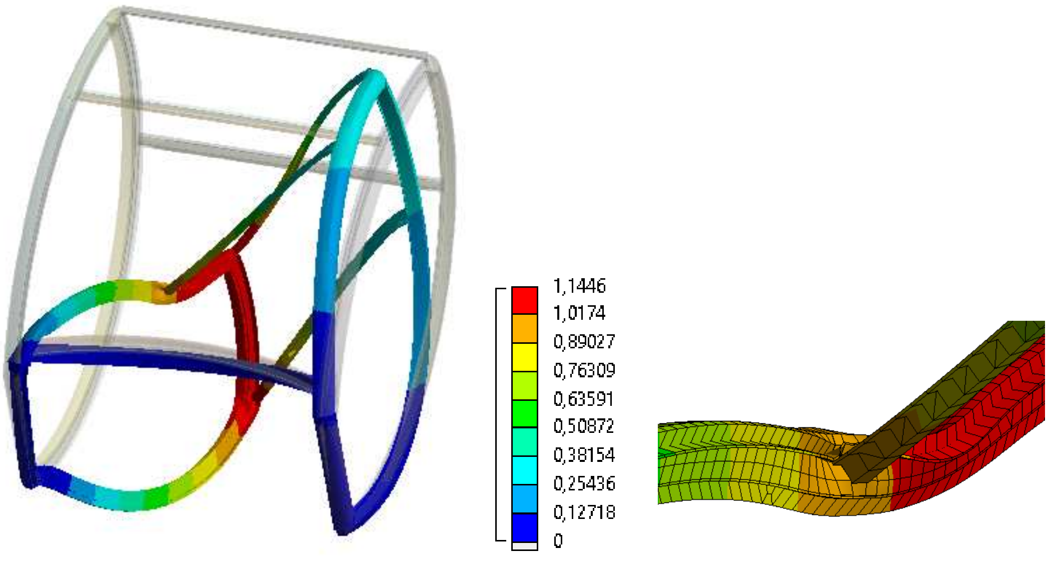

Figure 12. Deformed shape for shell element results. Units in [m]. The second image (joint between the lateral and roof beams) is used for Displacement plots.

In Figure 12 there is a post-collapse deformed shape of the structure. The figure was selected in order to show how the different types of deformation involved, showing a coupling between bending, torsion and plastic hinge in both efforts, coupled with buckling and warping.

This example shows a very high complex type of deformation, in which practically all phenomena occurring at the beam are coupled. This is typical of a catastrophic collapse beyond any design point.

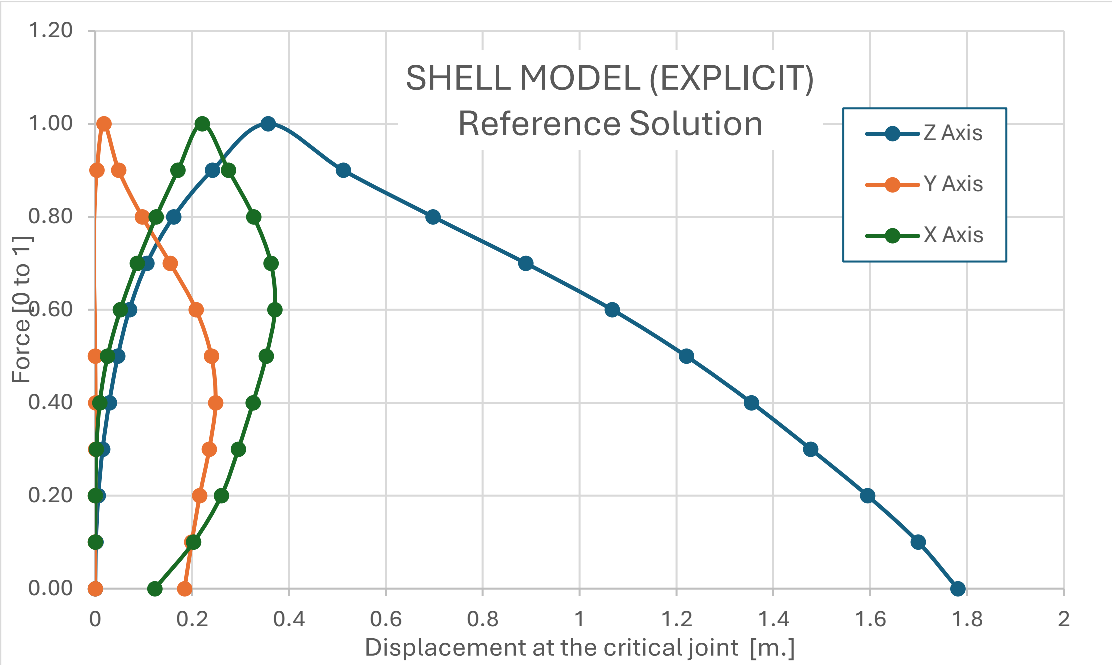

Figure 13. Equilibrium Paths (force vs. Displacement) at the point connecting lateral and roof beams. Force axis is scaled Force=1 indicates 30,000 N. max load.

In Figure 13, the plots represent the paths for non-linear behaviour of a complete structure, but the path has the same phases of the theoretical analyses shown in the previous sections. An important note is that the boundary conditions and loads do not represent any Certification test in particular. This way, this analysis is for research purposes only.

These results obtained from the shell model will be used to analyze the further results made with beam elements.

8.2. Beam element chassis models

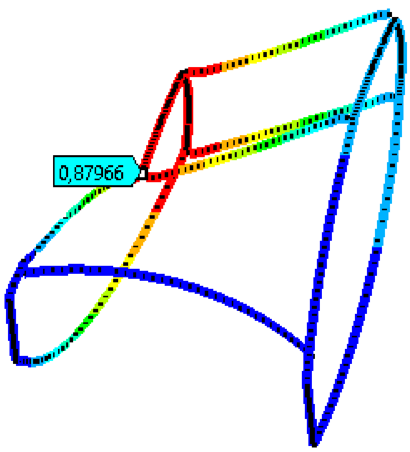

Figure 14. Total Displacements in Beam elements. Units in [m].

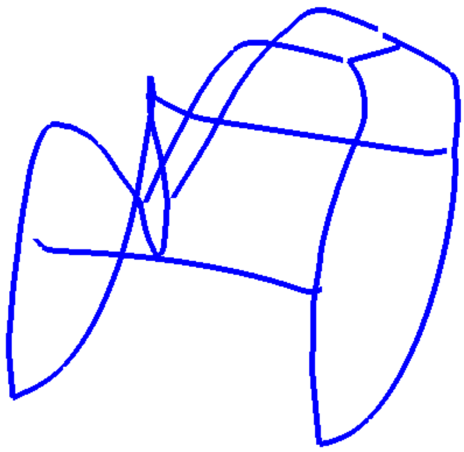

Figure 15. Explicit Beam Resistance Model. Deformed Shape [no Units].

In Figure 14 and Figure 15 we can appreciate the obvious difference between running the explicit model with Beam Resistance Model (no coupling between bending and torsion) and a pure beam explicit model, with material nonlinearity and Reissner formulation. The above model is very close to the shells, and the latter has collapse mechanisms that are not representative of the ones seen to this point. This will also be appreciated more in detail in the plots in Figure 16 and Figure 17.

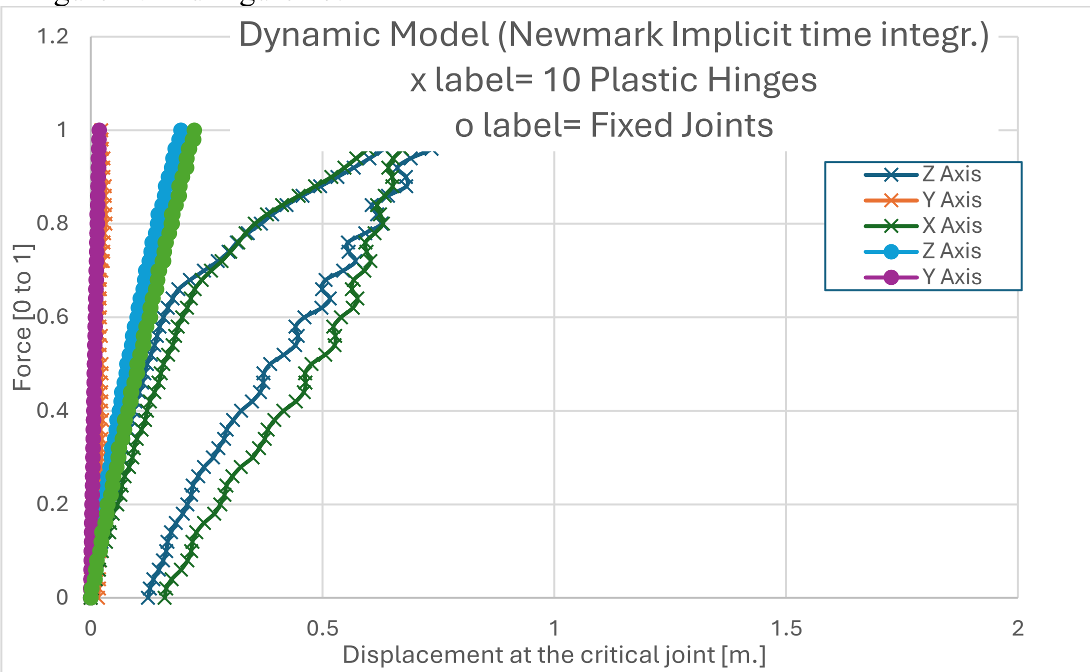

Figure 16. Plots for models with Newmark Time Integration

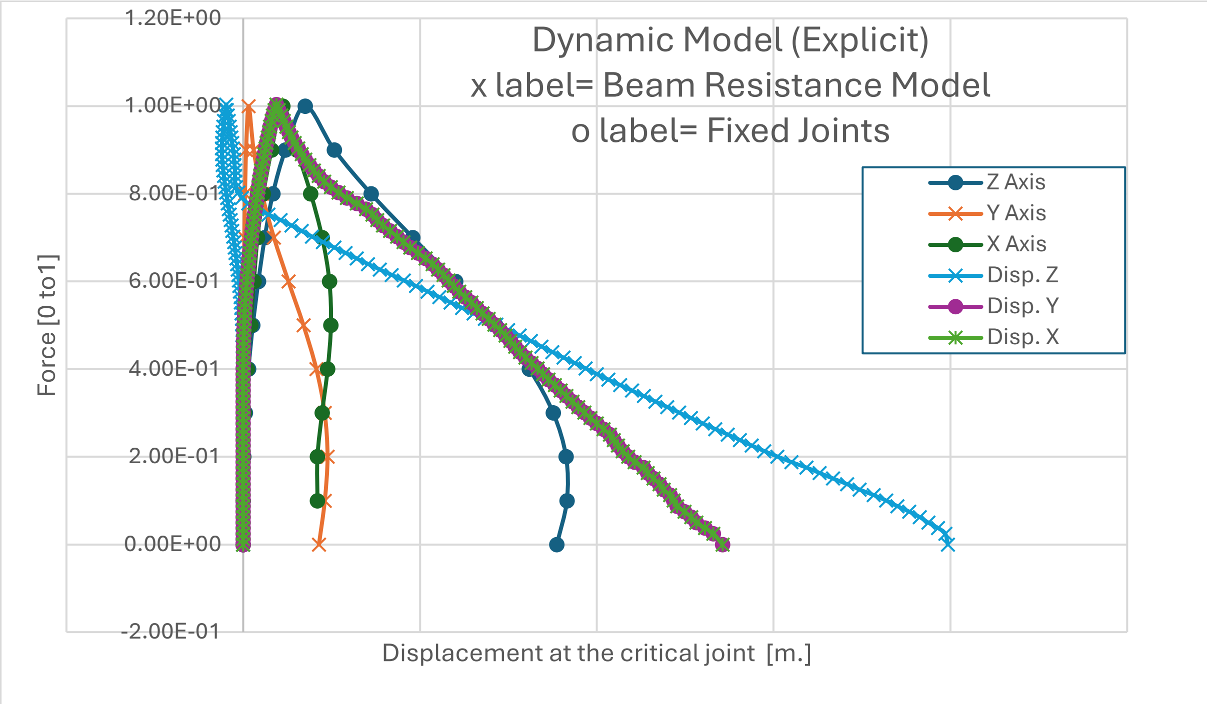

Figure 17. Plots for comparison between Beam Resistance Model and a Model with beams only.

9. DISCUSSION

In this work, the modelling of a primary chassis structure was analyzed in order to investigate the collapse behaviour of a vertically applied dynamic load. The chassis consisted of a frame structure with a general section, submitted to collapse loading conditions. Several modelling approaches have been used, and different time integration schemes.

In the first phase, the shell elements were correlated with theoretical models, including physical experiments. Due to a lack of physical test in the chassis, this correlation was used as a base reference for comparison.

In the second phase, all types of beam finite elements were tested in order to determine whether the simplification from shell elements to beam finite elements was viable.

First, the static analyses had shown the correlation between the static and dynamic plots in the plastic hinge mechanisms. The Strain Rate Effect, combined with the plastic hinges in bending and torsion, had proven to change the plot keeping the static shape approximately between the 0.5 and 5 s. range. Before 0.5 s. the dynamic effects are so strong that the collapse mechanism is no longer a plastic hinge, and when the time of application of the ramp load is above 5 s. the tendency is to approximate with the static model.

Within this framework, the results in Newmark time integration are showing a springback that do not match with the results in the shell model.

In the case of Explicit results, both beam elements based on Reissner Beam theory and "Beam resistance model" in Autodyn had been tested. The latter is a model where the Moment-Rotation curves are entered for 11, 22 and 33 directions, which is bending and torsion nonlinear behaviour in the beams constitutive model:

$$G = \frac{E}{2(1+\nu)} \tag{3}$$

$$T = \frac{JG}{L}\theta \tag{4}$$

In these two equations, the Beam Resistance model inputs nonlinear behaviour for both bending and torsion. This nonlinear expressions act at G and J properties, according to the curves that had been input. This effect is at the node level on the entire model, not at a certain joint nodes chosen by the user, like in the case of hybrid models that combine beam and nonlinear spring elements.

In the Beam Resistance Model equations, the bending and torsion sifnesses are not cupled, which means that the results are considerably different from the shell models. In the latter, the effects of nonlinear plastic hinge in bendig, coupled with nonlinear Saint-Venant torsion, altogether coupled with nonuniform torsion, and buckling, are shown.

10. CONCLUSSIONS

Due to the high complexity of working with a beam general section, submitted to all kinds of nonlinearities in bars, bending and torsion, the results for beam finite elements are inconclusive. As shown in the plots, there has not been found a beam model that represents with high precision the real collapse of the framed chassis, being the Explicit model with only beams the closest (but not conclusive) model. On the other hand, the shell modelling of the car frame is considered acceptable, because of the basis in correlation with physical tests, specially in bending.

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