Asociación Argentina
de Mecánica Computacional

Mecánica Computacional Vol XLII, pp. 495-504
A. Caggiano, G. Etse, P. Folino, M. Goldschmit, M. Pucheta, M. Storti (Eds.)
V. Cortínez, J. Inaudi, O. Moller (Issue eds.)
Buenos Aires, November 11-14, 2025

MOVING MESH MODELLING IN THE FREQUENCY DOMAIN FOR INDUCED VIBRATION ESTIMATION OF VEHICLES ON BEAMS SUPPORTED ON ELASTIC MEDIA

José A. Inaudi[a,b]

[a]*Facultad de Ingeniería, Universidad Católica de Córdoba , Argentina,*
https://www.ucc.edu.ar/

[b]*Facultad de Ciencias Exactas Físicas y Naturales, Universidad Nacional de Córdoba, Argentina,*
jose.antonio.inaudi@unc.edu.ar, https://fcefyn.unc.edu.ar

Keywords: Vibrations, finite elements, elastic media, moving mesh, frequency domain

Abstract. Moving one-dimensional Bernoulli beam elements in the frequency domain are presented in this paper for applications on induced vibrations due to moving loads or vehicles in contact with rails or roads supported by elastic homogeneous media. The model can be used for dynamic response estimation of trains on railroad tracks or vehicles on roads. A conventional motionless finite-element strategy requires very large meshes to allow the estimation of induced vibration of a moving vehicle, because a large portion of the mesh is required to model the distance travelled by the vehicle during simulation, in addition to domains at both sides of the model to develop adequate boundary conditions. An alternative approach is the use of a moving mesh so that the vehicle does not approach the boundaries of the model; this determines a significant reduction of the mesh size. The moving mesh moves at the speed of the vehicle, maintaining the contact-points at fixed locations in the moving reference frame. This approach leads to a time-invariant model for constant velocity vehicle or moving load in the case of a homogeneous foundation. In this paper, the moving beam and foundation model is developed in the frequency domain, computing the dynamic stiffness matrix of moving beam elements on a visco-elastic foundation. In addition, random process modelling of roughness of the rails or road allows the assessment of its effect on induced vibration of moving vehicles on infinite media. Different vehicle models can be connected the moving mesh model, including different number of wheel axes by defining nodes of the mesh bellow each wheel, making the formulation very practical. Some application examples of the proposed modelling technique are presented and limitations of the technique are mentioned.

ISSN: 2591-3522 DOI: 10.70567/mc.v42.ocsid8549

1 INTRODUCTION

Accurate models are required to predict ground-borne vibration due to moving vehicles on roads. Vibrations are caused by several excitation mechanisms, such as moving contact, moving loads, wheel imperfections and road unevenness, among other reasons. Several authors have studied moving loads or masses on infinite elastic domains (Steele, 1967; Fryba,1999; Anderson et al., 2001). Analytical solutions for the dynamic response of an infinite beam resting on a viscoelastic foundation and subjected to arbitrary dynamic loads have been developed by Yu and Yuang, 2014 among other authors. Since domains are usually infinite, a typical finite-element (FE) mesh models a portion of the domain and incorporates appropriate absorbing boundary conditions to emulate the behavior of an infinite domain with the finite model domain. In the case of moving loads, moving mass in contact with the domain, or moving vehicle on a road, the estimation of induced vibration may require large domains so that the moving contact/load stays within the mesh limits for the simulation time at the assumed velocity of the contact. This requires large meshes and substantial computational effort that can be overcome by using moving finite elements in the time domain (Inaudi, 2024).

In this paper a very efficient approach in the frequency domain is followed to assess both vibrations due to moving loads, moving masses or moving vehicles at constant speed on elastic foundations, and also, the stochastic component of induced vibrations due to road roughness modelled as a random stationary process. The use of dynamic stiffness matrices of finite elements with moving mesh in the frequency domain is a more precise and computationally more efficient method to handle this type of problems than conventional finite-elements in the time domain (Inaudi, 2024). The method requires that the modelled foundation domain be homogeneous and allows the consideration of representing multiple moving loads, moving masses or moving vehicles in contact with the elastic domain at a finite number of points that maintain relative distance (see Figure 1). The main advantage of the frequency domain approach with respect to the time domain approach (Inaudi, 2024) is that large portions of the domain between contact points and the semi-infinite domains at both sides can be modelled with single elements without compromising accuracy and allowing for a very efficient treatment of boundary conditions.

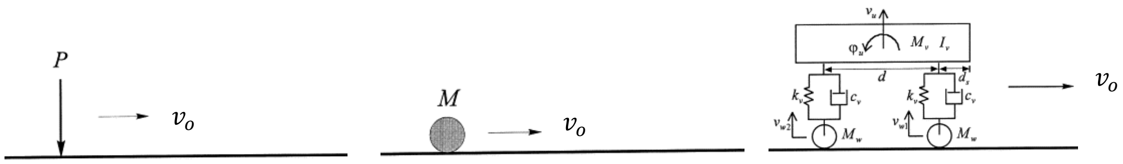

Figure 1. Problems of moving load, mass or vehicle on infinite elastic media.

The moving mesh is conceived in a moving reference frame (relative coordinates), modelling the displacement fields using relative coordinates. The speed of the moving frame is that of the moving load, moving mass or moving vehicle (v_o).

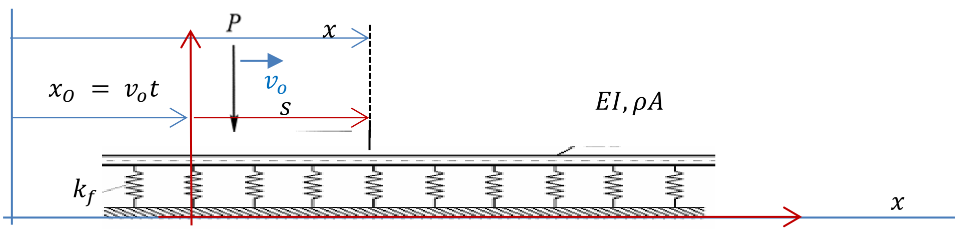

Figure 2. Moving load on a beam on elastic foundation.

The vertical displacement $u(x,t)$ of the beam section located at a distance x from a fixed frame of reference (as that depicted in light blue in Figure 2) can be defined in a moving frame of reference at constant velocity v_o(as that depicted in red in Figure 2) using a relative coordinate s, such that $s = x - v_o\, t$, defining a function $r(s,t)$:

$$u(x,t) = r(s,t) = r(x - v_o\, t, t) \tag{1}$$

The vertical velocity of the section can be expressed in terms of the field $r(s,t)$ as

$$\frac{\partial u(x,t)}{\partial t} = \frac{\partial r(s,t)}{\partial s}\frac{ds}{dt} + \frac{\partial r(s,t)}{\partial t} = -\frac{\partial r(s,t)}{\partial s} v_o + \frac{\partial r(s,t)}{\partial t} \tag{2}$$

since $\frac{ds}{dt} = -v_o$. On the other hand, because $\frac{ds}{dx} = 1$, the curvature of the Bernoulli beam for small deformations can be computed as

$$\frac{\partial^2 u(x,t)}{\partial x^2} = \frac{\partial}{\partial x}\left(\frac{\partial r(s,t)}{\partial s}\frac{ds}{dx}\right) = \frac{\partial}{\partial x}\left(\frac{\partial r(s,t)}{\partial s}1\right) = \frac{\partial^2 r(s,t)}{\partial s^2}\frac{ds}{dx} = \frac{\partial^2 r(s,t)}{\partial s^2} \tag{3}$$

Equations (2) and (3) allow the definition of the kinetic energy and elastic potential energy of a moving mesh (moving reference frame) to derive de differential equations of motion in the field $r(s,t)$ applying the Hamilton variational principle. The kinetic energy of a portion of the beam of length L (neglecting rotational inertia of the beam sections) can be expressed as:

$$T = \frac{1}{2}\int_0^L \rho A \left(-\frac{\partial r(s,t)}{\partial s} v_o + \frac{\partial r(s,t)}{\partial t}\right)^2 ds \tag{4}$$

where ρ and A are the density and the cross section of the beam. Eq. (4) indicates that the kinetic energy of a continuum model discretized by interpolation functions in relative coordinates in a constant velocity reference frame, leads to an expression of kinetic energy with three terms: a quadratic form of time-derivatives of the nodal displacements, a linear form of the time-derivatives of the nodal displacements, and a quadratic for of the nodal displacements.

On the other hand, because curvature (as Eq. (3) indicates) or strains in a general finite-element model with linear kinematics do not involve a change in the differential operator on $r(x,t)$ because $\frac{ds}{dx} = 1$, the elastic potential energy cab be expressed as:

$$U_e = \frac{1}{2}\int_0^L EI \left(\frac{\partial^2 r(s,t)}{\partial s^2}\right)^2 ds + \frac{1}{2}\int_0^L k\, r(s,t)^2 ds \tag{5}$$

The partial differential equation of a moving Bernoulli beam on elastic foundation with no external load applied on the domain can be obtained using Hamilton principle:

$$\rho A \frac{\partial^2 r(s,t)}{\partial t^2} - 2v_o \rho A \frac{\partial^2 r(s,t)}{\partial s\, \partial t} + {v_o}^2 \rho A \frac{\partial^2 r(s,t)}{\partial s^2} + EI \frac{\partial^4 r(s,t)}{\partial s^4} + kr(s,t) = 0 \tag{6}$$

If distributed viscous dissipation forces are assumed acting on the foundation, the differential equation results

$$\rho A \frac{\partial^2 r(s,t)}{\partial t^2} - 2v_o \rho A \frac{\partial^2 r(s,t)}{\partial s\, \partial t} + {v_o}^2 \rho A \frac{\partial^2 r(s,t)}{\partial s^2} + EI \frac{\partial^4 r(s,t)}{\partial s^4} + c\left(\frac{\partial r(s,t)}{\partial t} - v_o \frac{\partial r(s,t)}{\partial s}\right) + kr(s,t) = 0 \tag{7}$$

2 MOVING BEAM FINITE ELEMENT IN THE FREQUENCY DOMAIN

In this section we derive the dynamic stiffness of a finite-length (l) moving-beam finite element assuming complex exponential nodal displacements and rotations in the boundaries

$$r(0,t) = u_1\, e^{j\varpi t}, \frac{\partial r(0,t)}{\partial s} = u_2\, e^{j\varpi t}, r(l,t) = u_3\, e^{j\varpi t}, \frac{\partial r(l,t)}{\partial s} = u_4\, e^{j\varpi t} \tag{8}$$

where $j = \sqrt{-1}$.

We look for the stationary solution of Eq. 7 for these boundary conditions, assuming

$$r(s,t) = \psi(s)\, e^{j\varpi t} \tag{9}$$

and finally compute the harmonic shear forces, $V(s,t)$, and moments, $M(s,t)$, to be applied at the boundaries for the assumed harmonic displacement fields

$$F_1 e^{j\varpi t} = -V(0,t), M_1 e^{j\varpi t} = -M(0,t),\ F_3 e^{j\varpi t} = V(l,t),\ \ F_4 e^{j\varpi t} = M(l,t) \tag{10}$$

Where $j = \sqrt{-1}$. Replacing Eq. (9) in Eq. (7) we can obtain the forth-order homogeneous differential equation for $\psi(s)$at a given forcing frequency ϖ:

$$\psi''''(s) + a_3\psi'''(s) + a_2\psi''(s) + a_1\psi'(s) + a_o\psi(s) = 0 \tag{11}$$

where the coefficients a_i have the following expressions:

$$a_o = \frac{-\rho A\varpi^2 + k + j\varpi c}{EI},\quad a_1 = \frac{-2v_o\rho A j\varpi - c v_o}{EI},\quad a_2 = \frac{{v_o}^2\rho A}{EI},\quad a_3 = 0 \tag{12}$$

To compute the solution we write Eq. (11) in state-space as a first-order differential equation:

$$\frac{dz}{ds} = \boldsymbol{A}\, \boldsymbol{z}(s) \tag{13}$$

Where the matrix $\boldsymbol{A}$ and the space vector $\boldsymbol{z}$ are defined as:

$$\boldsymbol{z}(s) = \begin{bmatrix} \psi(s) \\ \psi'(s) \\ \psi''(s) \\ \psi'''(s) \end{bmatrix} \qquad \boldsymbol{A} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ a_o & a_1 & a_2 & a_3 \end{bmatrix} \tag{14}$$

Because matrix $\boldsymbol{A}$ is constant (independent of s) the solution of this space-invariant linear homogeneous differential equation can be expressed as:

$$\psi(s) = C_1 e^{\beta_1 s} + C_2 e^{\beta_2 s} + C_3 e^{\beta_3 s} + C_4 e^{\beta_4 s} \tag{15}$$

where β_i $(i = 1,2,3,4)$ are the four non-repeated eigenvalues of $\boldsymbol{A}$.

Applying the four kinematic boundary conditions defined in Eq. (8) and considering Eq. (9) we can express:

$$\psi(0) = u_1\,,\ \psi'(0) = u_2\,,\ \psi(l) = u_3\,, \psi'(l) = u_4 \tag{16}$$

From Eqs. (15) and (16):

$$\boldsymbol{B}_c \begin{bmatrix} C_1 \\ C_2 \\ C_2 \\ C_2 \end{bmatrix} = \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \text{ where } \boldsymbol{B}_c = \begin{bmatrix} 1 & 1 & 1 & 1 \\ \beta_1 & \beta_2 & \beta_3 & \beta_4 \\ e^{\beta_1 l} & e^{\beta_2 l} & e^{\beta_3 l} & e^{\beta_4 l} \\ \beta_1 e^{\beta_1 l} & \beta_2 e^{\beta_2 l} & \beta_3 e^{\beta_3 l} & \beta_4 e^{\beta_4 l} \end{bmatrix} \tag{17}$$

Finally, considering that $V(s,t) = -EI\frac{\partial^3 r(s,t)}{\partial s^3}$ and $M(s,t) = EI\frac{\partial^2 r(s,t)}{\partial s^2}$, from Eq. (10) and Eq. (17):

$$\begin{bmatrix} F_1 \\ M_2 \\ F_2 \\ M_2 \end{bmatrix} = \boldsymbol{H} \begin{bmatrix} C_1 \\ C_2 \\ C_2 \\ C_2 \end{bmatrix} \text{ where } \boldsymbol{H} = EI \begin{bmatrix} {\beta_1}^3 & {\beta_2}^3 & {\beta_3}^3 & {\beta_4}^3 \\ -{\beta_1}^2 & -{\beta_2}^2 & -{\beta_3}^2 & -{\beta_4}^2 \\ -{\beta_1}^3 e^{\beta_1 l} & -{\beta_2}^3 e^{\beta_2 l} & -{\beta_3}^3 e^{\beta_3 l} & -{\beta_4}^3 e^{\beta_4 l} \\ {\beta_1}^2 e^{\beta_1 l} & {\beta_2}^2 e^{\beta_2 l} & {\beta_3}^2 e^{\beta_3 l} & {\beta_4}^2 e^{\beta_4 l} \end{bmatrix} \tag{18}$$

Finally,

$$\begin{bmatrix} F_1 \\ M_2 \\ F_2 \\ M_2 \end{bmatrix} = \boldsymbol{H}\boldsymbol{B}_c^{-1} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \tag{19}$$

that can be written in terms of the stiffness matrix, $\boldsymbol{S}_b(\varpi)$, of the beam on elastic foundation:

$$\begin{bmatrix} F_1 \\ M_2 \\ F_2 \\ M_2 \end{bmatrix} = \boldsymbol{S}_b(\varpi) \begin{bmatrix} u_1 \\ u_2 \\ u_3 \\ u_4 \end{bmatrix} \text{ where } \boldsymbol{S}_b(\varpi) = \boldsymbol{H}\boldsymbol{B}_c^{-1} \tag{20}$$

Consider a moving constant vertical load $-P$ at constant speed on an infinite beam on elastic foundation. To estimate the stationary response of the domain, a simple 3-node model with two elements (Fig. 3) can be used to assemble the structural model dynamic stiffness matrix. To approximate infinite domain we can define sufficiently large elements at both sides of the central node on which the external moving load is applied. Because the load is constant in magnitude, the stiffness matrices are evaluated for $\varpi \to 0$. Therefore

$$\lim_{\varpi \to 0} \boldsymbol{S}_b(\varpi) \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ q_4 \\ q_5 \\ q_6 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ -P \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

Consider now that the moving load $P(t)$ is modelled as a stationary random process with one-sided power spectral density (psd) $G_{PP}(\varpi)$. The frequency response function from the moving load to displacements $\boldsymbol{q}(t)$ can be expressed as

$$H_{\boldsymbol{q}P}(\varpi) = \boldsymbol{S}_g(\varpi)^{-1} \begin{bmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{22}$$

The psd of the stationary response of the structural model can be computed as

$$G_{\boldsymbol{qq}}(\varpi) = H_{\boldsymbol{q}P}(\varpi)^* G_{PP}(\varpi) H_{\boldsymbol{q}P}(\varpi)^T \tag{23}$$

If a vehicle model is moving at constant velocity v_o on the beam, the mass, damping, and stiffness matrices of the vehicle can be assembled at the structural level by extending the generalized coordinate vector of the beam with the generalized coordinates of the vehicle model and assembling to the extended model the contribution to the dynamic stiffness matrix. This can be easily computed using the mass, damping and stiffness matrices of the vehicle model and computing its dynamic stiffness matrix as

$$S_v(\varpi) = -\varpi^2 M_v + j\varpi C_v + K_v \tag{24}$$

To simplify the assembly process, the beam mesh in relative coordinates must have nodes defined at the contact points of the vehicle wheels on the beam. In the following subsection some applications of moving load and moving vehicle are presented.

If a moving mass in direct contact with the beam on elastic foundation (BoEF) is to be modelled, we can simply assemble the corresponding dynamic stiffness of the moving mass to the vertical displacement coordinate of the beam mesh on elastic foundation. The following 2x2 dynamic stiffness of the moving mass can be computed associated to the transverse displacement and rotation of the node in which the mass is assembled (Inaudi, 2024)

$$S_m(\varpi) = \begin{bmatrix} -m\varpi^2 & 2jmv_o\varpi + mv_o^2 \\ 0 & 0 \end{bmatrix} \tag{25}$$

Analysis of multiple-axes moving vehicle with roughness contact on a BoEF

Consider a multiple-axis vehicle as that show in Figure 3.

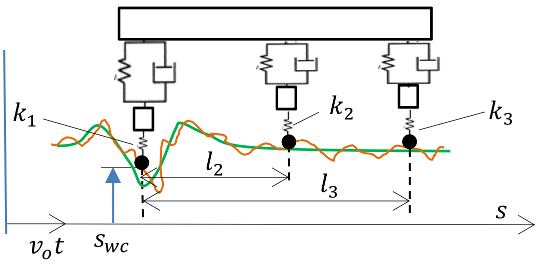

Figure 3. Vehicle model on elastic media with roughness in contact.

Generalized forces acting on moving nodes (contacts of moving vehicles and moving beam) can be easily computed for the generalized vehicle coordinates $\boldsymbol{q_v}$, and the generalized beam-node coordinates, $\boldsymbol{q_b}$, considering the elastic potential energy of the elastic element that models the wheel contact:

$$U = \frac{1}{2}\sum_{i=1}^{N_c} k_i \left(q_{vi} - q_{bi} - w_i(t)\right)^2 \tag{26}$$

Where N_c is the number of elastic contacts. Therefore the generalized non-conservative force resulting from road roughness can be expressed as

$$\frac{\partial U}{\partial q_{vj}} = k_j\left(q_{vj} - q_{bj} - w_j(t)\right) \qquad \frac{\partial U}{\partial q_{bj}} = -k_j\left(q_{vj} - q_{bj} - w_j(t)\right) \tag{27}$$

The linear terms on q_{vj} and q_{bj}can be assembled directly in the dynamic stiffness matrix at a structural level. The terms proportional to the $w_j(t)$ can be included as non-conservative terms in the right hand side of the equations of motion of the model as:

$$Q_{vj}(t) = k_j w_j(t) \qquad Q_{bj}(t) = -k_j w_j(t) \tag{28}$$

Assuming the contact nodes are numbered backwards from 1 to N_a, in the direction opposite to v_o as shown in Figure 3, the delay between the signals can be expressed in terms of the distance l_j between the j-th contact and the first contact as:

$$w_j(t) = w_1\left(t + l_j/v_o\right) \tag{29}$$

In the frequency domain we can express the Fourier transform of these signals as:

$$W_j(\varpi) = e^{-j\varpi l_j/v_o} W_1(\varpi) \tag{30}$$

This means that we can formulate the problem with a single-input model ($w_1(t)$) in the frequency domain. The cross power-spectral density for signals $w_i(t)$ and $w_n(t)$ can be expressed as:

$$G_{w_i w_n}(\varpi) = e^{j\varpi l_i/v_o} G_{w_1 w_1}(\varpi) e^{-j\varpi l_j/v_o} \tag{31}$$

where the formula can be generalized for any indices i and n, considering that $l_1 = 0$.

On the other hand, if spatial roughness is defined as a stationary random process, $R(x)$, in spatial coordinate (longitudinal distance), at constant speed, the time process $w_1(t)$ is defined as:

$$w_1(t) = R(v_o t) \tag{32}$$

Therefore, if the one-sided psd of $R(x)$ is $G_{RR}(\Omega)$, where Ω is the spatial frequency, the psd of $w_1(t)$ can be expressed as:

$$G_{w_1 w_1}(\varpi) = \frac{G_{RR}(\varpi/v_o)}{v_o} \tag{33}$$

Using this formulation a random process model of road roughness can be incorporated to assess the stochastic response of the foundation and multiple-axis vehicle moving at constant velocity.

3 SIMPLE EXAMPLE CASES

Two brief application examples of beam-FE in the frequency domain and moving coordinate system are presented in this section.

3.1 Stationary deformation for constant moving load

A comparison of the frequency-domain formulation and a time-domain formulation is done in this section. A 250 m beam on undamped elastic foundation with a moving constant load is analyzed. For the frequency domain analysis, a 3-node 2-element model is analyzed (with two elements of 125 m and the constant load in the central node). For the time-domain analysis (Inaudi, 2024) a finite-element mesh of 250 elements and 251 nodes separated by FE of 1 m in length is created to analyze the stationary response of the beam on elastic foundation subjected to constant moving load (shown in Figure 3). The parameters considered for this example are $A = 1;\ \rho = 1;\ L = 1;\ E = 100000;\ v_o = 60;\ I = 1;\ P = -98.1;\ k_f = 100,\ c_f = 0.$ The stationary response to an external constant applied in the central node of the mesh (shown in light blue in Figure 4) is computed. The figure in the right shows in blue line the vertical displacement field, $r_{st}(s)$, computed with the model matrices assembled at a structural model level (251×2 dofs) solving for the particular time-independent solution q_p of the ODE system (Inaudi, 2024):

$$(\boldsymbol{K}_b + \boldsymbol{K}_f + \boldsymbol{K}_{cf} + \boldsymbol{H})\, \boldsymbol{q}_p = \boldsymbol{P} \tag{34}$$

where $\boldsymbol{K}_b$, $\boldsymbol{K}_f$, $\boldsymbol{K}_{cf}$ and $\boldsymbol{H}$ are the elastic beam, elastic foundation, viscous foundation and moving mesh mass contributions to the structural-model pseudo-stiffness matrix for the moving mesh model (see Inaudi 2024 for details). In dotted red line the estimated displacement field using the frequency domain model is depicted. As expected, minor differences are observed. It is worth mentioning that the system of algebraic equations to solve in the case of the frequency-domain 3-node model is of 6x6, while in the case of the conventional time-domain 151-node FE model is of 302x302.

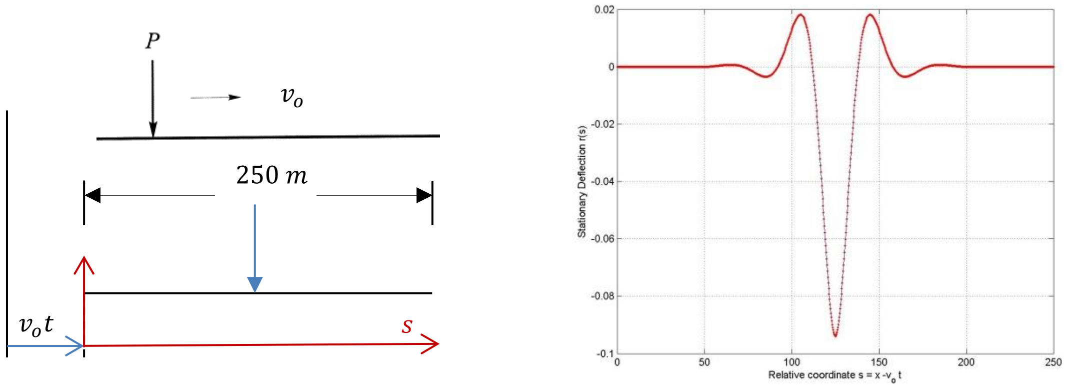

Figure 4. Stationary deformation field $r_{st}(s)$ for single vertical load (blue lines) and for the same vertical load applied in two nodes (magenta lines)

3.2 Analysis of 1-axis moving vehicle with roughness in contact

To illustrate the application of moving mesh to the analysis of a moving vehicle on a beam on elastic foundation, including roughness in contact, a simple model is developed with a spring k_n and no viscous damper in the contact between vehicle and beam. Figure 5 shows the mechanical parameters of the model and the definition of the generalized displacements.

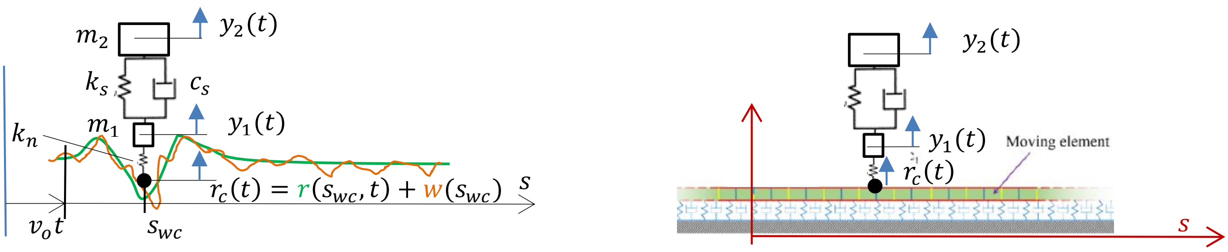

Figure 5. Quarter vehicle model on elastic media with roughness in contact.

If roughness of the wheel or road surface is to be included in to assess its influence in vibration of the road-vehicle system, the vertical displacement of the wheel contact can be expressed as:

$$r_c(t) = r(s_{wc}, t) + w(v_o t + s_{wc}) = L_{wc}\, \boldsymbol{q}(t) + w(v_o t + s_{wc}) \tag{35}$$

where s_{WC} is the relative coordinate of the node of wheel contact under consideration, L_{wc} is the kinematic transformation from FE nodal displacements to $r(s_{wc}, t)$, and $w(v_o t + s_{wc})$ is the roughness vertical displacement model (deterministic or random).

To construct the model of the vehicle with the vertical displacements $r_c(t)$, $y_1(t)$, and

$y_2(t)$ the mass, damping and stiffness matrices of the vehicle model (see Figure 4) are assembled:

$$M_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & m_1 & 0 \\ 0 & 0 & m_2 \end{bmatrix} \qquad K_v = \begin{bmatrix} k_n & -k_n & 0 \\ -k_n & k_n + k_s & -k_s \\ 0 & -k_s & k_s \end{bmatrix} \qquad C_v = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_s & -c_s \\ 0 & -c_s & c_s \end{bmatrix} \tag{36}$$

The dynamic stiffness of the vehicle (Eq. 24) can be assembled to the full dynamic stiffness of the structural model of the beam and vehicle.

Figure 6 shows the power spectral density of the contribution of $w(t)$ to the vertical displacement ($q_3(t)$) of the contact beam node of the vehicle model. The assumed vehicle parameters are $m_1 = 0.1$, $m_2 = 1$, $k_s = 600$, $k_n = 2400, c_s = 15$. The roughness psd is assumed as:

$$S_{RR}(\Omega) = S_{RR}(\Omega_o)\,(\frac{\Omega}{\Omega_o})^{-2} \tag{37}$$

where Ω_o =0.1 1/m y $S_{RR}(\Omega_o) = 1/m^3$.

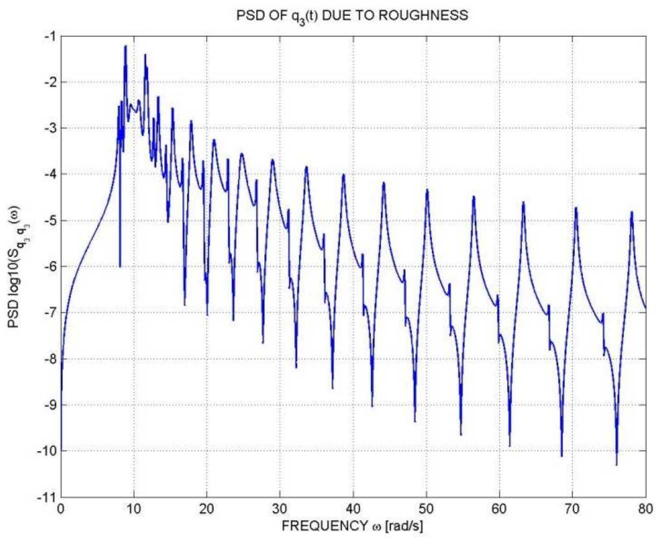

Figure 6. PSD of roughness contribution to vertical displacement of vehicle-beam contact node.

4 CONCLUSIONS AND FURTHER RESEARCH

The development of frequency-domain elements for the estimation of response of moving vehicles or moving loads on elastic homogeneous infinite domains has been presented. The use of moving meshes (formulation of displacement fields in relative coordinates) allows the construction of versatile computational models for the estimation of vehicle-induced vibrations with different applications, requiring a significantly smaller mesh than that of conventional stationary finite elements. These tools can be applied for vibration-intensity estimation for environmental impact analysis of train or vehicle induced vibrations, including road or rail roughness using random vibration analysis. Automation in model generation for vehicles consisting in multiple cars (for train applications) moving at constant velocity on elastic rails will be approached in the near future as an extension of this work. Other lines for future research are *i*) the feasibility of an homogenization strategy of periodic substructures such as sleepers under rails so that the proposed formulation can approximate the mechanical behavior moving vehicles on rails supported by sleepers and other periodic substructures using a moving mesh formulation, *ii*) a strategy for approximating the response of moving vehicles on non-homogeneous soil domains with stochastic elastic properties, *iii*) the relevance of incorporating appropriate boundary layers in the moving mesh to allow absorbing boundaries, and *iv*) the use of other linear viscoelastic damping models in the frequency domain could be considered and would not significantly change the formulation of dynamic stiffness developed in this paper.

ACKNOWLEDGEMENTS

This piece of research has been supported by the Catholic University of Córdoba and the Science and Technology Agency (SECyT) of the province of Córdoba as part of a study on vibration generation and propagation on non-homogeneous soil for application in environmental impact analyses of induced vibrations.

REFERENCES

Andersen L., Nielsen S. R., and Kirkegaard P., "Finite element modelling of infinite Euler beams on Kelvin foundations exposed to moving load in convected co-ordinates," *Journal of Sound and Vibration*, vol. 241, no. 4, pp. 587–604, 2001.

Fryba L., *Vibration of Solids and Structures under Moving Loads*, Springer Science and Business Media, Berlin, Germany, 3 edition, 1999.

Inaudi J.A, "Moving Finite-Element Mesh Modelling for Induced Vibration Estimation of Moving Vehicles on Infinite Elastic Media," *Mecánica Computacional* Vol XLI, pp. 33-44 C.I. Pairetti, M.A. Pucheta, M.A. Storti, C.M. Venier (Eds.) S. Ferreyra, M. Sequeira, R. O'Brien (Issue eds.) Rosario. Asociación Argentina de Mecánica Computacional, 2024.

Steele C., "The finite beam with a moving load," *Journal of Applied Mechanics*, vol. 34, no. 1, pp. 111–118, 1967.

Yu, H. and Yuan Y., "Analytical Solution for an Infinite Euler-Bernoulli Beam on a Viscoelastic Foundation Subjected to Arbitrary Dynamic Loads," *Journal of Engineering Mechanics*, Vol. 140, No. 3, March 1, 2014.