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A NOVEL PHYSICALLY-BASED INTERPOLATION METHOD FOR BATHYMETIC DATA

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Abstract. Accurate Digital Terrain Models (DTMs) are essential for reliable two- and three-dimensional hydrodynamic and sediment transport modeling in rivers. These DTMs are typically derived from survey data, but full coverage of the modeling domain is rarely achieved. As a result, interpolation—or even extrapolation—of bathymetric data becomes necessary. Common interpolation methods, such as Inverse Distance Weighting (IDW) or Triangular Irregular Networks (TIN), are often used but can introduce unphysical artifacts that compromise model accuracy. This paper presents a novel physically-based interpolation technique for bathymetric data, called Velocity-Based Diffusion (VBD). The method involves solving the Shallow Water Equations (SWE) to obtain a tentative flow field, which is then used as input for a custom partial differential equation (PDE) solver implemented in OpenFOAM to propagate the surveyed bathymetric data. The process is applied iteratively: the flow field is updated using the latest bathymetry, and the bathymetry is updated using the new solution from the solver. Although computationally intensive, this approach produces artifact-free DTMs with minimal user intervention, making it particularly well-suited for river modeling applications.

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1 INTRODUCTION

River bathymetry is one of the most influential inputs in hydrodynamic and morphodynamic modeling. The geometry of the channel controls flow distribution, velocities, turbulence structures, and sediment transport pathways. However, in practice, survey data are typically collected along a limited number of transects or points due to time, cost, and logistical constraints. This sparse coverage creates the need for interpolation methods to reconstruct a continuous Digital Terrain Model (DTM) of the riverbed.

Traditional interpolation techniques have been extensively applied, but their limitations become evident in fluvial environments. Methods such as IDW, TIN or Kriging rely purely on geometric relationships among data points and do not incorporate hydraulic principles. As a result, they may generate unrealistic features such as spurious depressions, ridges, or disconnected flow pathways. Moreover, their applicability to extrapolation (e.g., outside the surveyed corridor) is extremely limited and generally produces unphysical results. Other specialized methods are applicable to specific conditions, and are difficult to generalize. A short description of the most common methods is presented in the next section.

To address these challenges, this work proposes a physically-based interpolation framework that uses hydrodynamic principles as the organizing mechanism for spreading bathymetric information across the domain. By combining numerical solutions of the Shallow Water Equations (SWE) with diffusion-based interpolation guided by the resulting velocity field, the method naturally aligns reconstructed bathymetry with the river's flow structure.

1.1 Common Interpolation Methods

In this section, a short description of the most common interpolation methods for river bathymetric data is given. For a more detailed description, see Bello-Pineda and Stefanoni-Hernández (2007), Curtarelli et al (2015), and Andes and Cox (2017).

Inverse Distance Weighting (IDW)
A deterministic method where values are estimated as weighted averages of nearby samples, with weights inversely proportional to distance. While simple and fast, IDW often produces circular artifacts around survey points and disregards anisotropy in flow-oriented domains.

Anisotropic Inverse Distance Weightings
An extension of IDW where the weighting function accounts for directional biases. This reduces some artifacts but the selection of the directional biases remains purely empirical. The river axis is usually given extra interpolation weight. Several variants of this method exist, such as EIDW and RIDW.

Triangular Irregular Networks (TIN)
TIN methods construct a mesh of non-overlapping triangles between survey points and interpolate values within each triangle. This provides exact reproduction of measured data but introduces sharp discontinuities and may generate unrealistic slopes when survey spacing is irregular. It performs best if the surveys are spread evenly over the domain, which is seldomly the case in fluvial applications.

Kriging
Kriging is a geostatistical interpolation technique based on spatial correlation models. It assumes that the bathymetric field can be represented as a stochastic process, with covariance structures estimated through variograms. This allows Kriging to provide both an interpolated value and an associated error estimate. Although flexible and widely used in geosciences, its application to rivers is constrained by the need for dense survey data to calibrate variograms, and by the lack of explicit consideration of flow

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dynamics.

Morphology-based methods.

These approaches rely on the assumption that river cross-sectional shapes remain consistent along a reach. Data must be given in the form of cross sections, so not all surveys are appropriate. The surveyed profiles are interpolated longitudinally, and intermediate bathymetry is generated by smoothly transitioning between cross-sections. These methods are best suited for schematic studies or regions where river geometry is relatively uniform, and do not handle bifurcation or confluences. The HEC-RAS modelling software includes tools to apply this interpolation method to 1D models (Hydrologic Engineering Center, 2023).

2 PROPOSED METODOLOGY

2.1 Outline

The proposed methodology, termed *Velocity-Based Diffusion* (VBD) interpolation, combines hydrodynamic and diffusion-based PDE solvers to interpolate bathymetric data. Figure 1 illustrates the main steps:

1. The computational domain is meshed.
2. A flat bathymetry is prescribed as the initial guess.
3. The Shallow Water Equations are solved to obtain a preliminary velocity field.
4. A diffusivity field is derived as a function of local velocities.
5. A diffusion equation is solved, propagating surveyed bathymetric values across the domain.
6. The interpolated bathymetry replaces the initial guess.
7. The process is repeated iteratively until convergence, alternating between SWE and diffusion solvers.

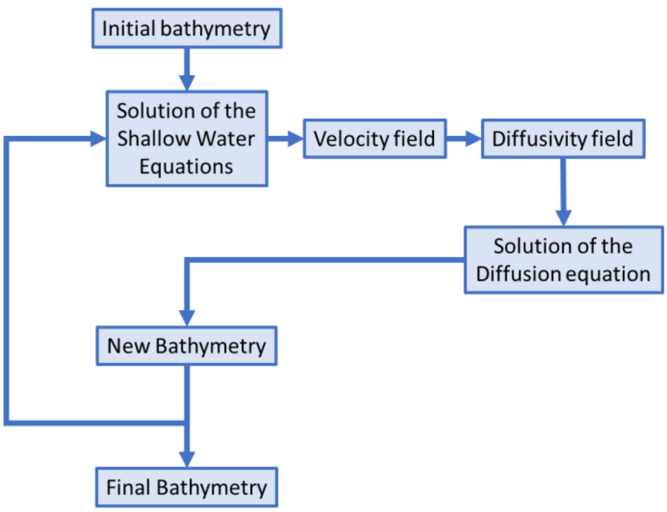

Figure 1: Outline of the interpolation algorithm.

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This iterative coupling ensures that interpolated bathymetry evolves consistently with the hydraulic behavior of the system. The iterative nature of the algorithm requires a certain convergence criterion to be defined, either in terms of the velocity field or the bathymetric levels not changing between successive iterations above a certain threshold. This convergence has not been studied in detail yet, but it was observed in practice that the algorithm tends to produce very little changes after the third iteration, providing the initial guess is reasonable.

2.2 Shallow Water Equations

The hydrodynamic backbone of the method is the two-dimensional depth-averaged Shallow Water Equations (SWE), expressed as:

$$\frac{\partial h}{\partial t} + \nabla \cdot (hU) = 0$$

$$\frac{\partial (hU)}{\partial t} + \nabla \cdot \left(hU \otimes U + \frac{1}{2} g h^2 I \right) = -gh\nabla z_b - \tau_b$$

where h is water depth, U is depth-averaged velocity, z_b is bed elevation, g is gravitational acceleration, and τ_b represents bottom friction.

The equations are solved using the TELEMAC 2D (Hervouet, 2007) system, which provides robust finite element/finite volume solvers for free-surface flows in natural rivers. A transient solution of the flow, starting from approximate initial conditions is computed until a steady state solution is reached.

For best results, a good estimate of the water level should be used as downstream initial condition, and a good estimate of the average flow rate on the river reach should be used as upstream boundary condition.

2.3 Diffusion Equations for interpolation

A diffusion equation is solved, propagating surveyed bathymetric values across the domain. The velocity field is used to define a diffusivity tensor that controls how survey points influence the surrounding grid. The governing PDE has the following expression:

$$\frac{\partial z_b}{\partial t} - \nabla \cdot (D \nabla z_b) = S$$

where D is the anisotropic diffusivity tensor, and S is a source term, that imposes the effect of the bathymetric surveys.

The source term in the equation is responsible for representing the bathymetric surveys. It has the following formulation:

$$S = Mask\, K(z_{b0} - z_b)$$

where z_{b0} is the elevation of the surveys. This source term only operates in cells in which surveyed data exists. This is enforced by a mask term:

$$Mask = \begin{cases} 1 & if\ survey\ available\ in\ cell \\ 0 & otherwhise \end{cases}$$

The diffusivity tensor is anisotropic and variable point to point, based on the velocity field obtained from the Shallow Water Equations. Since the velocity field is defined in a per-node basis in TELEMAC, the field is interpolated to cell centers before computing the diffusivity. The first principal axis is aligned with the velocity vector, and hence the longitudinal direction, while the second one (transversal) is

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aligned with the conjugate direction. The diffusivity along each axis has a constant component A. Additionally, the longitudinal component has a component that increases linearly with the flow velocity:

$$D = S\,M\,S^{-1}$$

$$S = \begin{bmatrix} \cos(\alpha) & -\sin(\alpha) & 0 \\ \sin(\alpha) & \cos(\alpha) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M = \begin{bmatrix} \mathrm{A} + \mathrm{B}\|U\| & 0 & 0 \\ 0 & \mathrm{A} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\alpha = atan(\frac{U_2}{U_1})$$

Different values for the A, B and K parameters were tested. Ultimately, the following set was used in all the cases presented in this paper:

$$A = 10\ m^2/s$$

$$B = 100{,}000\ m$$

$$K = 100\ 1/s$$

These values worked well in a range of cases. However, no attempt to systematize their choice has been attempted yet, and hence it is entirely possible that the choice is suboptimal. Additionally, values might also need to be different for cases of markedly different scales (e.g., much larger or smaller rivers). Adimensionalization of the parameters could help mitigate this issue.

By aligning and scaling the diffusivity with flow direction and magnitude, surveyed data are preferentially spread along streamlines rather than across them, yielding hydraulically consistent interpolations.

This PDE is solved using a custom solver called *dibathyFoam*, built using the OpenFOAM framework (Weller et al, 1999). OpenFOAM allows great flexibility for the implementation and iterative modification of PDE solvers.

Dirichlet conditions are applied at the river banks, setting the bottom elevations to be equal to the free surface elevation estimated for the river, which can be considered uniform for short river reaches, or have a longitudinal profile imposed for longer reaches.

For the inlet and outlet boundaries, Neumann (zero-gradient) conditions are applied.

The Diffusion Equation is solved in transient mode until a stationary solution is found. It was found that it is important discretized the source term implicitly for stability reasons. It is also necessary to apply several non-orthogonal correction steps in order to properly compute the diffusive fluxes in triangular meshes. The Laplacian is discretized with the Gauss Linear Corrected scheme.

Bed elevations obtained from the Diffusion Equation solver are defined on the cell centers of the mesh. Before being reused in TELEMAC they must be interpolated to the nodes. This interpolation introduces some additional numerical diffusion, which can be mitigated by using a fine discretization mesh. In the future, a custom code could be developed allowing the diffusion and shallow water equations could be solved on the same mesh, thus removing this issue entirely.

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3 APPLICATIONS

Over the past year, the proposed methodology has been applied in several engineering studies along the Paraguay river, with overall good results.

A short description of some of these applications is presented below.

3.1 Río Paraguay downstream of Urbano

The interpolation methodology was applied to a 12.5 Km reach of the Paraguay river located downstream of the town of Urbano (Paraguay). Approximately 2,200 points have been previously surveyed in the reach, and were available as a (x,y,z) coordinated set.

The reach was discretized into a mesh of approximately 170,000 triangular elements, with typical sized between 10 and 25 m. Initially a constant bathymetric level of 42 mIGN, with a flow rate of 1,670 m3/s and a downstream water level of 48.16 mIGN. Two iterations of the interpolation methodology were then performed, in each case computing first the flow solution with the best available bathymetric data, and then solving the diffusion equation to interpolate the surveyed data using the derived diffusivity field. A constant level of 47.16 mIGN was imposed as boundary condition on the river banks for the diffusivity equation.

A general and a zoomed in view of the survey data and the interpolated bathymetry are presented on Figure 2. Notice how the survey data is not necessarily arranged to form defined cross sections. This is not an issue, since the algorithm treats the surveys as a general cloud of dispersed point data. Despite this limitation on the source data, the interpolated field still properly resolves the continuous features of the bottom. The presence of relatively high depths on the outer size of the meander, near the right bank (flow goes from north to south in this reach) is especially apparent. Notice how some extrapolation is performed with the method near the north boundary without any visible artifacting.

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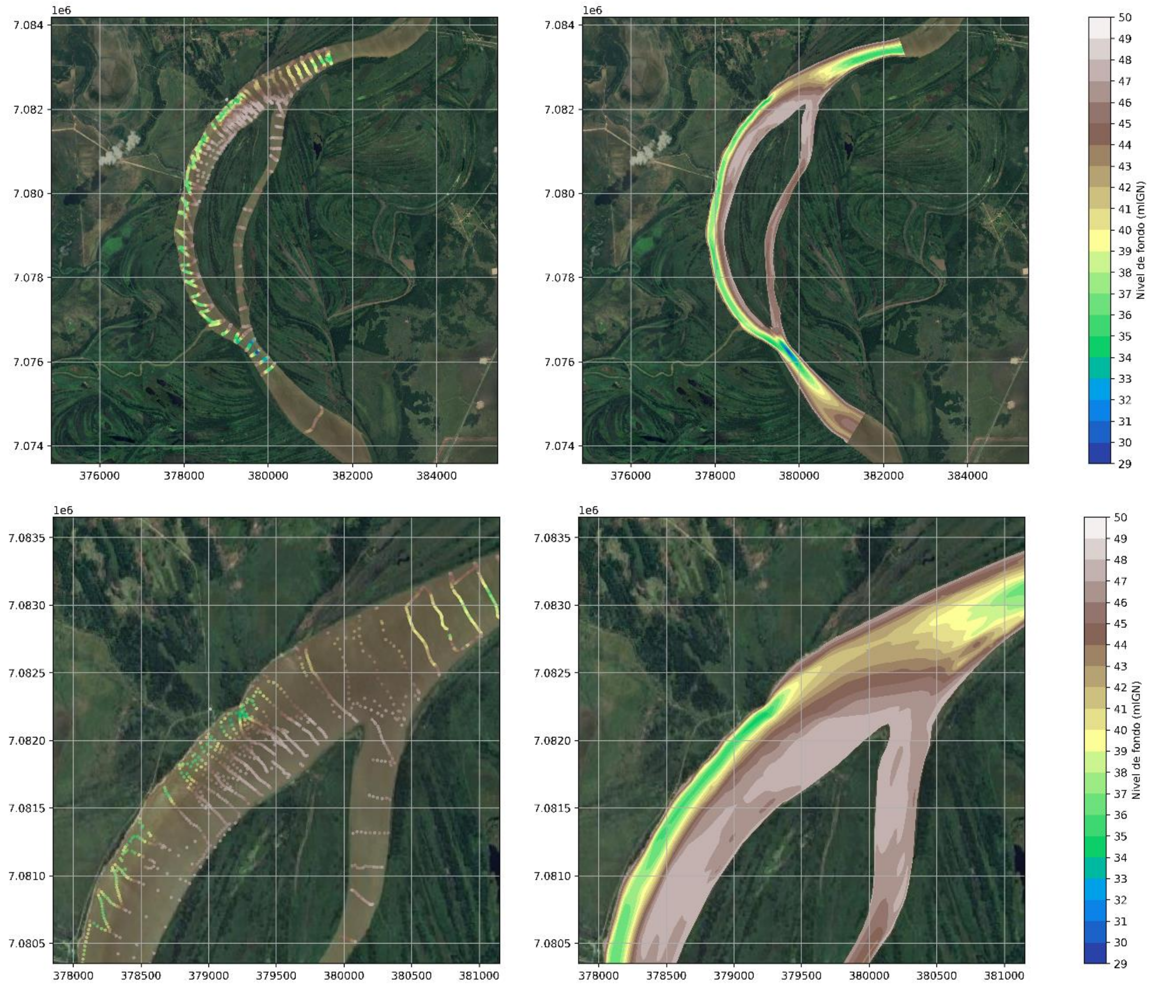

Figure 2: Surveyed and interpolated data for the first application case.

3.2 Río Paraguay downstream of Pilar

The method was also applied to an 8.5 Km reach of the Paraguay River, located just downstream of Pilar (Paraguay). This reach includes the confluence with the Bermejo River. In this case different bathymetries were constructed for different dates, since the river experiences significant morphological evolution on this site. On this paper, the one constructed for July 2025 is presented. Approximately 5,400 points were surveyed on the site during that period.

The reach was discretized into a mesh of approximately 150,000 triangular elements, with typical sized between 5 and 10 m. A constant bottom level of 42 mIGM was used as initial seed, with flow rates of 3,750 m3/s in the Paraguay River and 700 m3/s in the Bermejo, and a downstream water level of 49.80 mIGM. Three iterations of the interpolation methodology were performed in this case, with a constant level of 49.80 mIGM imposed as boundary condition on the river banks for the diffusivity equation.

Different views of the surveyed and the interpolated data are presented on Figure 3.

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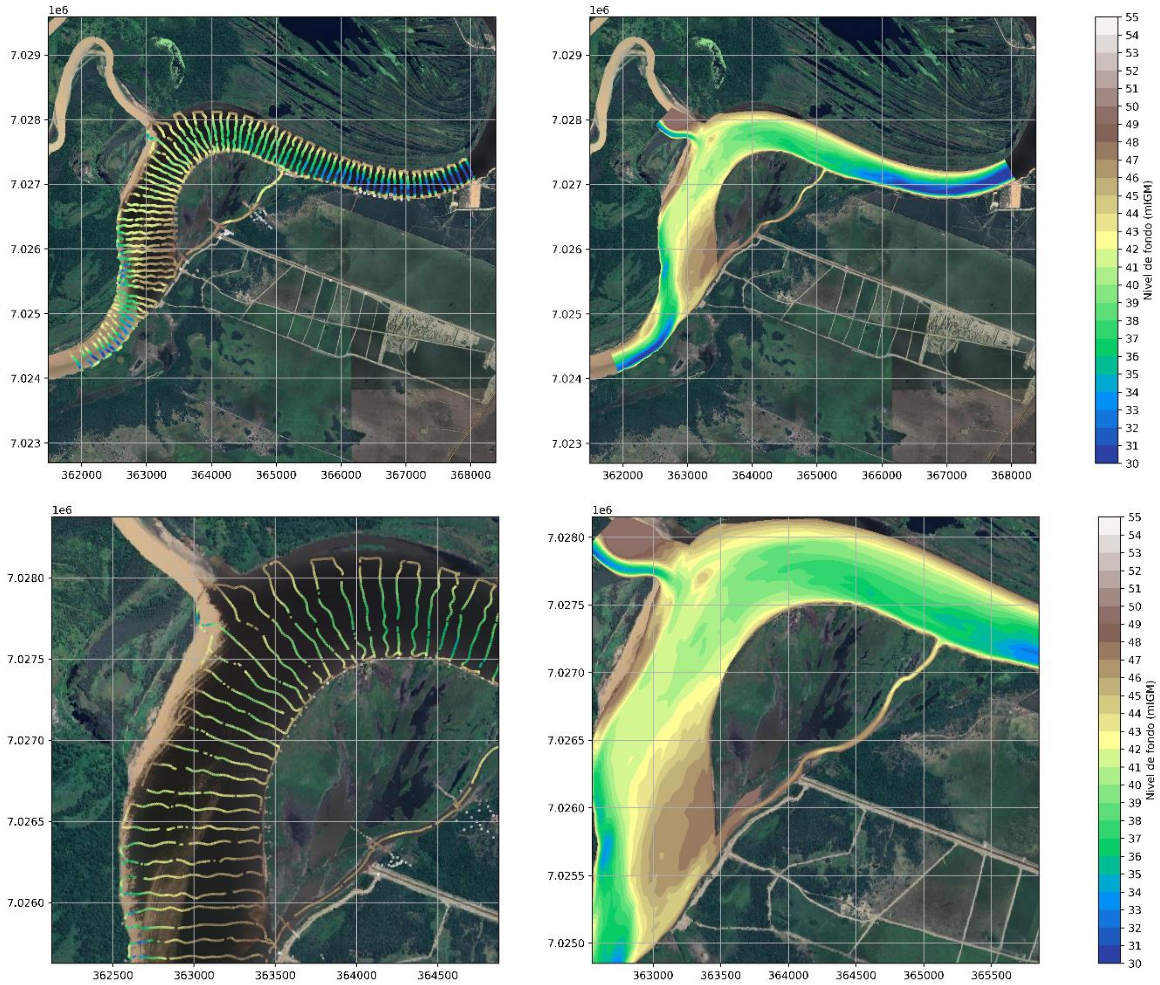

Figure 3: Surveyed and interpolated data for the second application case.

3.3 Upper Río Paraguay

Finally, the application to a much longer reach of the Upper Paraguay River is presented. In this case, a bathymetric DEM was created for a 500 Km reach, extending from 70 Km upstream of Ladario (Brazil) to the confluence with the Apa River, near Vallemí (Paraguay). For this reach, a number of survey campaigns existed, which are a mix of cross section and longitudinal profiles data, totaling approximately 6 Million surveyed point arranged as (x, y, z) coordinates.

An outline of the river was downloaded from the OpenStreetMap dataset. In order to make the Telemac runs more manageable, the 500 Km reach was divided into 7 shorter sub-reaches. This decreased the computation time required on each model to reach steady state conditions. A triangular mesh was created for each one, with mesh sized of 10 m near the banks and islands, increasing to 25 m away from them. Total element count was in the range of 300,000 to 600,000 elements per model. An estimated free surface profile was imposed as bottom elevations on the banks, that was also used to impose level boundary conditions on the downstream end of each model. A constant flow rate of 1,200 m3/s was imposed on the upstream boundary of each model.

Two iterations of the interpolation methodology were applied in each subdomain. Figure 4 shows the view of some small reaches within the larger dataset. It is observed that the method generates a reasonable DEM even in otherwise difficult to resolve conditions, such the presence of multiple

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bifurcations and non-surveyed branches

It is also worth pointing out that a considerable time saving was achieved in this case by using this fully automated methodology instead of a more manual approach, which would have been very labor-intensive. Both the solution of the flow field with TELEMAC, and the solution of the diffusion equation with OpenFOAM were fully parallelized in this case. The entire solution of all the sub-reaches took approximately 8-10 hours on a Dual Xeon workstation with 24 total cores.

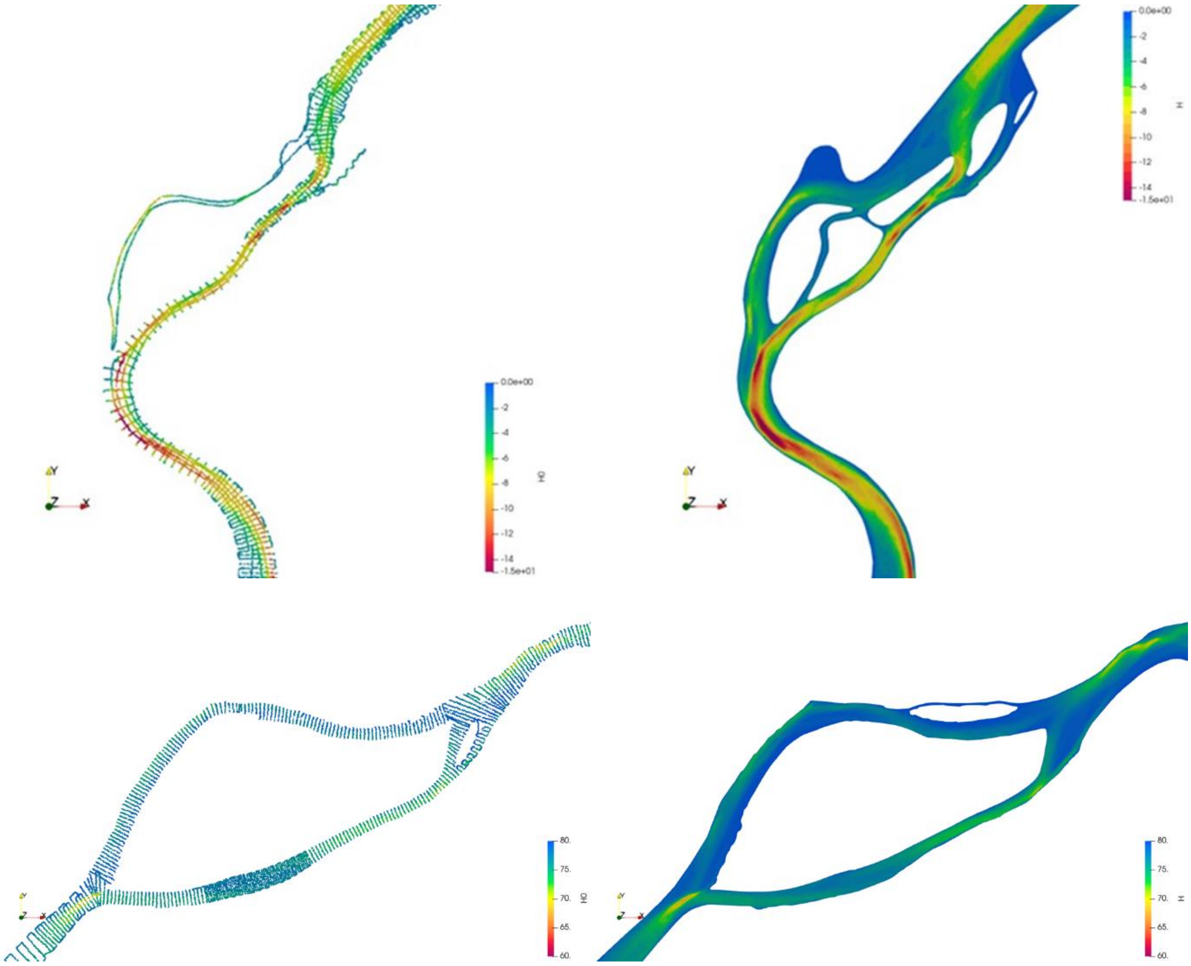

Figure 4: Detail view of some selected sites within reach interpolated for the third application case.

4 ANALYSIS OF PERFORMANCE

The performance of the proposed methodology was assessed using the case study described in Section 3.1, corresponding to a reach of the Paraguay River downstream of Urbano. The available survey dataset was randomly divided into two subsets: the source dataset, used to perform the interpolation, and the test dataset, used exclusively for error evaluation. Several source-to-test proportions were considered (90%/10%, 80%/20%, 50%/50%, and 10%/90%) to explore the sensitivity of the method to data availability.

The Velocity-Based Diffusion (VBD) method was applied to each of the reduced source datasets. For comparison, the commonly used IDW method (with exponent 2) and the TIN method were also applied to the same cases. Figure 5 illustrates the interpolated bathymetries for different levels of data reduction. It is evident that VBD produces substantially fewer artifacts than IDW and TIN, and that the

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relative advantage of VBD increases as the survey dataset becomes sparser.

Two complementary error metrics were employed. The first consisted of a point-by-point comparison between the interpolated surfaces and the test dataset. The resulting Root Mean Square Error (RMSE) and Mean Absolute Error (MAE) are reported in Table 1. On average, VBD reduced errors by 44% compared to IDW and by 13% compared to TIN.

Table 1: Comparison of the interpolated bathymetric field against the Test Dataset

Source Data - Test Data	RMSE (m)			MAE (m)		
	IDW	TIN	VBD	IDW	TIN	VBD
90% - 10%	1.25	0.73	0.67	0.90	0.48	0.45
80% - 20%	1.37	0.89	0.81	0.94	0.50	0.47
50% - 50%	1.51	0.99	0.90	1.03	0.57	0.53
10% - 90%	2.61	2.46	1.68	1.84	1.46	1.07

However, this first metric is influenced by the distribution of survey points, which are often closely spaced along cross sections. In such areas, IDW and TIN may appear competitive, while still producing unrealistic surfaces in unsurveyed regions. To address this limitation, a second error metric was introduced. In this case, interpolations generated from each reduced source dataset were compared against the interpolation obtained from the full dataset. This provides a measure of robustness, quantifying how sensitive each method is to reductions in data density.

The results of this second metric are presented in Table 2. Once again, VBD outperformed both IDW and TIN, with average errors 24% lower than IDW and 43% lower than TIN. These results demonstrate that VBD yields interpolated surfaces that are less sensitive to missing or sparse data, an important property for river bathymetry applications where surveys are typically incomplete.

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Figure 5: Comparison of the performance of IDW, TIN and VBD methods with different decimations of the source dataset.

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Table 2: Comparison of the interpolated field with decimated data against the interpolated field with full data

Source Data - Test Data	RMSE (m)			MAE (m)		
	IDW	TIN	VBD	IDW	TIN	VBD
90% - 10%	0.18	0.26	0.13	0.10	0.08	0.07
80% - 20%	0.27	0.43	0.20	0.16	0.18	0.11
50% - 50%	0.50	0.80	0.39	0.32	0.43	0.23
10% - 90%	1.76	2.94	1.61	1.25	1.91	1.01

Overall, the performance analysis confirms that the VBD approach not only produces smoother and more hydraulically consistent bathymetric surfaces, but also delivers superior accuracy and robustness compared to traditional interpolation methods, particularly under conditions of limited survey coverage.

5 CONCLUSIONS

This paper has presented a new physically based interpolation framework for river bathymetry. In contrast to conventional geometric methods, the proposed approach incorporates hydrodynamic information from the Shallow Water Equations to guide the spatial distribution of survey data. By coupling this with a diffusion-based PDE solver, the method produces smooth, artifact-free DTMs that remain consistent with river hydraulics.

Although more computationally demanding than traditional interpolation techniques, the method requires little manual intervention and scales effectively to large domains. Its ability to handle both interpolation and extrapolation makes it particularly suitable for river modeling applications where bathymetric accuracy is critical.

A quantitative performance assessment showed that the Velocity-Based Diffusion (VBD) method reduces RMSE by up to 44% compared to IDW and by 13% compared to TIN, demonstrating clear improvements in robustness and accuracy.

Future work will focus on better establishment the generality of the model parameters, and extending the framework beyond anisotropic diffusion. One promising direction is to couple sediment transport processes into the SWE solver and introduce erosion/deposition terms into the diffusion PDE, potentially allowing the method to infer deep or shallow zones even without direct survey data. Another line of development is to use systematic source/test dataset splits as a built-in calibration strategy, enabling automatic adjustment of method parameters either universally or on a case-by-case basis.

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