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EXACT AND APPROXIMATE ANALYSIS OF STRUCTURAL MODELS WITH LINEAR VISCOUS OR VISCOELASTIC DAMPERS AND SINGULAR MASS MATRICES

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Abstract. This research is concerned with model reduction methods in analysis of structural models with viscous or viscoelastic damping and singular mass matrices. These types of models are commonly used in the preliminary design of building structures containing supplemental damping devices to define locations and parameters of the energy dissipaters. In this paper we develop an exact reduced-order technique for the modal analysis of models with singular mass matrices and viscous or viscoelastic supplemental dampers. Using static condensation of generalized coordinates without mass and no damper connection, and using a transformation using the eigenvectors of the damping matrix, an exact state-space formulation with non-singular mass matrix is developed. Alternative approximate model-reduction strategies are revisited and the accuracy of these methods in the estimation of poles (natural frequencies and damping ratios) and frequency response functions is compared with that of the exact model.

Keywords: Dampers, viscoelasticity, structural dynamics, vibrations, order reduction.

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1 INTRODUCTION

When viscous or viscoelastic dampers are used to improve the dynamical performance of low-rise or high-rise structures, typically installed connecting adjacent floors using diagonal bracing or Chevron braces, damper forces are applied on both horizontal and vertical motions of connecting nodes. Is common practice in the preliminary design of structures with viscous or viscoelastic dampers to use reduced-order models and neglect mass in certain degrees of freedom of the main structure. Simplified models which neglect inertial forces in a subset of the degrees of freedom of the model determine a singular mass matrix.

Other typical cases of linear models with singular mass matrices arise when modeling viscoelastic elements created with Voight-Maxwell elements in parallel or Kelvin elements in series as shown in Figure 1.b.

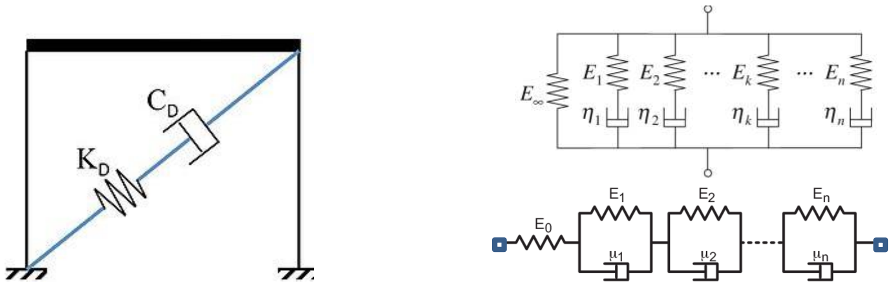

Figure 1. (a) Simple model of a frame with viscous dampers (b) Viscoelastic models

Static condensation of the stiffness matrix, Ritz vectors, or the modal strain energy method can be used to obtain approximate models for preliminary design (Inaudi et al., 1993). Static condensation of the stiffness matrix to generalized coordinates with non-zero associated mass can be used for model order reduction. However, if deformations of viscous or viscoelastic dampers depend on these statically condensed generalized coordinates, this condensation technique can introduce significant loss of accuracy in the reduced-order model. For this reason, in high-rise flexible structures, applying static condensation of all vertical nodal displacements is not a precise technique for model reduction. This aspect will be explored in this paper.

Load-dependent Ritz vectors is an approximate technique to obtain a reduced order model of a linear structure with viscous or viscoelastic dampers. The inclusion of damper force influence vectors along with the external load patterns, can provide increased accuracy in reduced order models (Léger and Wilson, 1987). The modal strain energy method is another alternative technique for the definition of location and parameters of supplemental viscous or viscoelastic dampers in high-rise structures subjected to wind forces or ground motions due to earthquakes (Inaudi et al., 1993). This method can provide approximate mode shapes, natural frequencies and damping ratios of the structural model with added linear dampers. The simpler form of this method assumes that the mode shapes of the undamped model are not changed by the dampers and simplifies the dynamics of the damped structure as uncoupled viscously damped classical modal second order differential equations for the generalized coordinates used in the formulation.

An exact formulation for order reduction of linear models with singular mass matrices and linear viscous supplemental dampers is presented. The formulation is an alternative to that presented by (Bath and Bernstein, 1996). The inaccuracy of approximate methods is analyzed. Using static condensation of generalized coordinates without mass and no damper connection, and using a transformation using the eigenvectors of the damping matrix, an exact state-space formulation with non-singular mass matrix is developed. Poles, natural frequencies and modal damping ratios are estimated in simple singular mass models to illustrate the proposed method. The above-mentioned alternative approximate model-reduction strategies are revisited and the accuracy of these methods in the estimation of poles (natural frequencies and damping ratios) and frequency response functions is compared with the exact model. To compare the accuracy of the alternative models developed a simple benchmark structural model with singular mass matrix is used.

2 MODELS WITH VISCOUS DAMPERS AND SINGULAR MASS MATRICES

A linear elastic structure with linear viscous dampers (VDs) and singular mass matrix exhibits symmetric mass, damping and stiffness matrices (M, C, and K). Condensing statically the stiffness matrix to maintain only those generalized coordinates with non-zero generalized mass or damping and reducing by static condensation all generalized coordinates with no mass associated and only generalized stiffness, the equations of motion of the structure with supplemental viscous dampers can be written as:

$$\begin{bmatrix} M_{dd} & O \\ O & O \end{bmatrix} \begin{bmatrix} \ddot{u}_d \\ \ddot{u}_v \end{bmatrix} + \begin{bmatrix} C_{dd} & C_{dv} \\ {C_{dv}}^T & C_{vv} \end{bmatrix} \begin{bmatrix} \dot{u}_d \\ \dot{u}_v \end{bmatrix} + \begin{bmatrix} K_{dd} & K_{dv} \\ {K_{dv}}^T & K_{vv} \end{bmatrix} \begin{bmatrix} u_d \\ u_v \end{bmatrix} = \begin{bmatrix} L_{dw} \\ O \end{bmatrix} w(t) \tag{1}$$

where u_d (t) are the generalized coordinates with associated non-singular mass matrix M_{dd}, and u_v (t) are those generalized coordinates with zero generalized mass and non-zero generalized viscous damping matrix.

Partitioning Eq. (1) we obtain:

$$M_{dd}\ddot{u}_d(t) + C_{dd}\dot{u}_d(t) + C_{dv}\dot{u}_v(t) + K_{dd}u_d(t) + K_{dv}u_v(t) = L_{dw}w(t) \tag{2}$$

$${C_{dv}}^T\dot{u}_d(t) + C_{vv}\dot{u}_v(t) + {K_{dv}}^T u_d(t) + K_{vv}u_v(t) = O \tag{3}$$

If C_{vv} is positive definite, its inverse exists and $\dot{u}_v(t)$ can be obtained from Eq. (3) as:

$$\dot{u}_v(t) = D\, u_v(t) + E_d u_d(t) + E_{dd}\dot{u}_d(t) \tag{4}$$

where

$$D = -{C_{vv}}^{-1} K_{vv},\ E_d = -{C_{vv}}^{-1} {K_{dv}}^T,\ \ E_{dd} = -{C_{vv}}^{-1} {C_{dv}}^T \tag{5}$$

Replacing Eq. (4) in Eq.(2) we obtain:

$$M_s\ddot{u}_d(t) + C_s\dot{u}_d(t) + K_s u_d(t) + L_s u_v(t) = L_{dw}w(t) \tag{6}$$

where

$$M_s = M_{dd},\ C_s = C_{dd} - C_{dv}{C_{vv}}^{-1}{C_{dv}}^T, K_s = K_{dd} - C_{dv}{C_{vv}}^{-1}{K_{dv}}^T, L_s = K_{dv} - C_{dv}{C_{vv}}^{-1}K_{vv} \tag{7}$$

Therefore, the dynamics of the model reduces to a set of second-order differential equations (Eq. (6)) coupled to a set of first-order differential equations (Eq. (4)). From Eq. (7) we note that K_s and L_s are not necessarily symmetric.

A suitable state vector for the system of ordinary differential equations defined in Eqs. (4) and (6) is:

$$x(t) = \begin{bmatrix} u_d(t) \\ \dot{u}_d(t) \\ u_v(t) \end{bmatrix} \tag{8}$$

Which leads to a standard first-order differential equation in state space

$$x(t) = A\,x(t) + B\,w(t) \tag{9}$$

Where the matrices A and B are

$$A = \begin{bmatrix} O & I & O \\ -M_s^{-1}K_s & -M_s^{-1}C_s & -M_s^{-1}L_s \\ E_d & E_{dd} & D \end{bmatrix}, \quad B = \begin{bmatrix} O \\ M_s^{-1}L_{dw} \\ O \end{bmatrix} \tag{10}$$

The mode shapes, natural frequencies and damping ratios of the system can be obtained by solving for the eigenvectors and eigenvalues (poles) of A and the response of the structure to transient excitation can be evaluated using various integration methods available for first-order differential equations. Modal superposition methods can be applied using eigenvectors of A as a basis and left eigenvectors of A to decouple modal coordinates.

Consider now the case of a singular symmetric matrix C_{vv}, i.e. ${C_{vv}}^{-1}$ does not exist. For this case, let us construct an orthogonal basis using the orthogonal eigenvectors of C_{vv} and define the following transformation:

$$u_v(t) = \Phi_1 q_1(t) + \Phi_2 q_2(t) \tag{11}$$

where Φ_1 is a matrix whose columns are the eigenvectors of C_{vv} associated with zero eigenvalues and Φ_2 is a matrix whose columns are the eigenvectors of C_{vv} associated with non-zero positive eigenvalues. This implies that we can express

$$C_{vv}\Phi_1 = O, \quad C_{vv}\Phi_2 = \Phi_2\Lambda \tag{12}$$

where Λ is a diagonal matrix containing with positive eigenvalues of $\mathrm{C_{vv}}$, $\Lambda(i,i) = \lambda_i$. Because C_{vv} is symmetric, the eigenvectors are orthogonal and can be normalized so that

$${\Phi_1}^T\Phi_1 = I, \quad {\Phi_2}^T\Phi_2 = I \tag{13}$$

where I is the identity matrix of the appropriate dimension. Using the transformation defined in Eq. (11) in Eq. (3) and pre-multiplying by ${\Phi_1}^{\mathrm{T}}$ we obtain

$$q_1(t) = -({\Phi_1}^T K_{vv}\Phi_1)^{-1}({\Phi_1}^T {C_{dv}}^T \dot{u}_d(t) + {\Phi_1}^T {K_{dv}}^T u_d(t) + {\Phi_1}^T K_{vv}\Phi_2 q_2(t)) \tag{14}$$

The equalities ${\Phi_1}^T C_{vv}\Phi_1 = 0$ and ${\Phi_1}^T C_{vv}\Phi_2 = 0$ eliminate $\dot{q}_1(t)$ and $\dot{q}_2(t)$ in Eq. (14).

Similarly, pre-multiplying Eq. (3) by ${\Phi_2}^{\mathrm{T}}$we obtain

$$\dot{q}_2(t) = -\Lambda^{-1}({\Phi_2}^T {C_{dv}}^T \dot{u}_d(t) + {\Phi_2}^T {K_{dv}}^T u_d(t) + {\Phi_2}^T K_{vv}\Phi_1 q_1(t) + {\Phi_2}^T K_{vv}\Phi_2 q_2(t)) \tag{15}$$

The equalities ${\Phi_2}^T C_{vv}\Phi_1 = 0$ and ${\Phi_2}^T C_{vv}\Phi_2 = \Lambda$ are used to eliminate $\dot{\mathrm{q}}_1(\mathrm{t})$ in Eq. (15).

Replacing Eq. (14) into Eq. (15) we obtain:

$$\dot{q}_2(t) = Dq_2(t) + E_d u_d(t) + E_{dd}\dot{u}_d(t) \tag{16}$$

where

$$\begin{aligned} D &= -\Lambda^{-1}[{\Phi_2}^T K_{vv}\Phi_2 - {\Phi_2}^T K_{vv}\Phi_1({\Phi_1}^T K_{vv}\Phi_1)^{-1}{\Phi_1}^T K_{vv}\Phi_2] \\ E_d &= -\Lambda^{-1}[{\Phi_2}^T {K_{dv}}^T - {\Phi_2}^T K_{vv}\Phi_1({\Phi_1}^T K_{vv}\Phi_1)^{-1}{\Phi_1}^T {K_{dv}}^T] \\ E_{dd} &= -\Lambda^{-1}[{\Phi_2}^T {C_{dv}}^T - {\Phi_2}^T K_{vv}\Phi_1({\Phi_1}^T K_{vv}\Phi_1)^{-1}{\Phi_1}^T {C_{dv}}^T] \end{aligned} \tag{17}$$

From Eq. (2) and using Eq. (11) we can write:

$$M_{dd}\ddot{u}_d(t) + C_{dd}\dot{u}_d(t) + C_{dv}\big(\Phi_1\dot{q}_1(t) + \Phi_2\dot{q}_2(t)\big) + K_{dd}u_d(t) + K_{dv}\big(\Phi_1 q_1(t) + \Phi_2 q_2(t)\big) = L_{dw}w(t) \tag{18}$$

Differentiating Eq. (14) with respect to time we obtain:

$$\dot{q}_1(t) = -(\Phi_1{}^T K_{vv} \Phi_1)^{-1}(\Phi_1{}^T C_{dv}{}^T \ddot{u}_d(t) + \Phi_1{}^T K_{dv}{}^T \dot{u}_d(t) + \Phi_1{}^T K_{vv} \Phi_2 \dot{q}_2(t)) \quad (19)$$

Finally, replacing Eq. (19) into Eq. (2) we obtain:

$$M_s \ddot{u}_d(t) + C_s \dot{u}_d(t) + K_s u_d(t) + L_s u_v(t) = L_{dw} w(t) \quad (20)$$

where

$$M_s = M_{dd} - C_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T C_{dv}^T$$

$$C_s = C_{dd} - C_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T \left(K_{dv}^T - K_{vv}\, \Phi_2 \Lambda^{-1} \Phi_2^T C_{dv}^T + K_{vv}\, \Phi_2 \Lambda^{-1} \Gamma_{12}^T \Gamma_{11}^{-1} \Phi_1^T C_{dv}^T \right) + \cdots$$
$$-C_{dv}^T \Phi_2 \Lambda^{-1} \left(\Phi_2^T C_{dv}^T - \Gamma_{12}^T \Gamma_{11}^{-1} \Phi_1^T C_{dv}^T - K_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T C_{dv}^T \right)$$

$$K_s = K_{dd} - C_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T \left(K_{vv}\, \Phi_2 \Lambda^{-1} \Phi_2^T K_{dv}^T \right) + \cdots$$
$$-C_{dv}^T \Phi_2 \Lambda^{-1} \left(\Phi_2^T K_{dv}^T - \Gamma_{12}^T \Gamma_{11}^{-1} \Phi_1^T K_{dv}^T) K_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T K_{dv}^T \right)$$

$$L_s = (-C_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T K_{vv} \Phi_2) \Lambda^{-1} (\Gamma_{22} - \Gamma_{12}^T \Gamma_{11}^{-1} \Phi_1^T K_{vv} \Phi_2) + \cdots$$
$$K_{dv}\, \Phi_1 \Gamma_{11}^{-1} \Phi_1^T K_{vv} \Phi_2 + K_{dv}\, \Phi_2 \quad (21)$$

where

$$\Gamma_{11} = \Phi_1^T K_{vv}\, \Phi_1 \quad \Gamma_{22} = \Phi_2^T K_{vv}\, \Phi_2 \quad \Gamma_{12} = \Phi_1^T K_{vv}\, \Phi_2 \quad (22)$$

The model can be formulated in state space as in Eq. (9) defining the following state vector and system matrices A and B:

$$x(t) = \begin{bmatrix} u_d(t) \\ \dot{u}_d(t) \\ q_2(t) \end{bmatrix} \qquad A = \begin{bmatrix} O & I & O \\ -M_s^{-1} K_s & -M_s^{-1} C_s & -M_s^{-1} L_s \\ E_d & E_{dd} & D \end{bmatrix} \qquad B = \begin{bmatrix} O \\ M_s^{-1} L_{dw} \\ O \end{bmatrix} \quad (23)$$

where

$$D = -\Lambda^{-1}[\Phi_2^T K_{vv} \Phi_2 - \Phi_2^T K_{vv} \Phi_1 (\Phi_1^T K_{vv} \Phi_1)^{-1} \Phi_1^T K_{vv} \Phi_2]$$

$$E_{dd} = -\Lambda^{-1}[\Phi_2^T C_{dv}^T - \Phi_2^T K_{vv} \Phi_1 (\Phi_1^T K_{vv} \Phi_1)^{-1} \Phi_1^T C_{dv}^T] \quad (24)$$

$$E_d = -\Lambda^{-1}[\Phi_2^T K_{dv}^T - \Phi_2^T K_{vv} \Phi_1 (\Phi_1^T K_{vv} \Phi_1)^{-1} \Phi_1^T K_{dv}^T]$$

The proposed formulation was coded as a Matlab® function to obtain the reduced-order model for symmetric damping matrices. In the following subsections some numerical examples are developed to test the formulation in simple cases and show the application in the estimation of modal damping ratios of a high-rise building structure with supplemental dampers.

2.1 Application of the proposed method to a simple structural model

To illustrate the application of the proposed analysis technique let us consider first a simple conceptual structural model with singular mass matrix and viscous dampers (Figure 2). The model can represent a single-story frame where k_d represents the condensed lateral stiffness, k_v the axial stiffness of the column and c the parameter of a linear viscous damper connecting the foundation and the first floor. The damper deformation depends on both lateral and vertical displacements $u_d(t)$ and $u_v(t)$. The simplified model considers a mass m_d for lateral displacement u_d and no mass associated to vertical displacement $m_v = 0$. The model mass, damping and stiffness matrices for small displacements are:

$$M = \begin{bmatrix} m_d & 0 \\ 0 & 0 \end{bmatrix},\ C = c \begin{bmatrix} \cos(\alpha)^2 & \cos(\alpha)\sin(\alpha) \\ \cos(\alpha)\sin(\alpha) & \sin(\alpha)^2 \end{bmatrix},\ K = \begin{bmatrix} k_d & 0 \\ 0 & k_v \end{bmatrix} \tag{26}$$

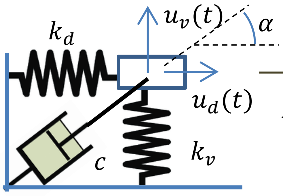

Figure 2. Simple model to illustrate the effect of axial flexibility of columns on the poles of an elastic frame with supplemental viscous dampers.

Even tough in a single-story frame the lateral stiffness is significantly smaller than the vertical stiffness of the column we define in this example a very flexible axial column to illustrate the effect of flexibility of long columns of high-rise buildings on modal damping augmentation using viscous dampers. Defining the parameters $m_d = 1, c = 5.0265, k_d = 39.4784, k_v = 39.4784$, and $\alpha = \pi/4$, the state space model obtained using the proposed formulation yields state-space matrices A and B:

$$A = \begin{bmatrix} 0 & 1 & 0 \\ -39.4784 & 0 & 39.4784 \\ 0 & -1 & -15.708 \end{bmatrix},\ B = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \tag{27}$$

The poles of the model can be computed as the eigenvalues of the state-space matrix A: $s_1 = -1.2146 + 6.7250i$ $s_2 = -1.2146 - 6.7250i$ $s_3 = -13.2788$ determining a natural frequency of 6.8338 rad/s and modal damping ratio of 0.1777 in the exact complex poles (s_1 and s_2).

Figure 3.a compares the frequency response function (FRF) form external horizontal load to lateral displacement computed with the reduced-order model

$$H_{u_d f_d}(\varpi) = [1 \quad 0 \quad 0](j\varpi I - A)^{-1}\mathrm{B} \tag{28}$$

where $j = \sqrt{-1}$, with the same FRF computed using the mass, damping and stiffness matrices of the model

$$H_{u_d f_d}(\varpi) = [1 \quad 0](-\varpi^2 M + j\varpi C + K)^{-1} \begin{bmatrix} 1 \\ 0 \end{bmatrix} \tag{29}$$

As shown in the figure the proposed technique provides the exact FRF for the model (both lines are superimposed in the left figure).

Figure 3.b compares the exact FRF computed with the proposed model with that of an approximate single-degree of freedom model that neglects vertical displacements (assumed infinite vertical stiffness k_v), that is

$$m_d \ddot{u}_d(t) + c\cos(\alpha)^2\, \dot{u}_d(t) + k_d u_d(t) = f_d(t) \tag{30}$$

As Figure 3.b shows, the single degree of freedom model that neglects vertical displacement (red line) underestimates the FRF amplitude close to resonance. The natural frequency of the simplified model is $\hat{\omega} = \sqrt{k_d/m_d} = 6.2832$ rad/s and the estimated damping ratio is $\hat{\xi} = c\cos(\alpha)^2/(2\, m_d\hat{\omega}) = 0.20$. This approximate model overestimates the exact damping ratio of the model ($\xi = 0.177$) and underestimates the natural frequency of the complex poles of the exact model ($\omega = 6.8338$ rad/s). This simple model illustrates the effect observed in high-rise buildings with supplemental viscous dampers connected in diagonals: if axial flexibility of columns is neglected, modal damping ratios can be overestimated with significant error in high-rise buildings with supplemental viscous dampers. Additionally, the axial flexibility of columns in high-rise buildings determines an upper limit to the modal damping ratios of the viscously damped structure (as we illustrate in section 3).

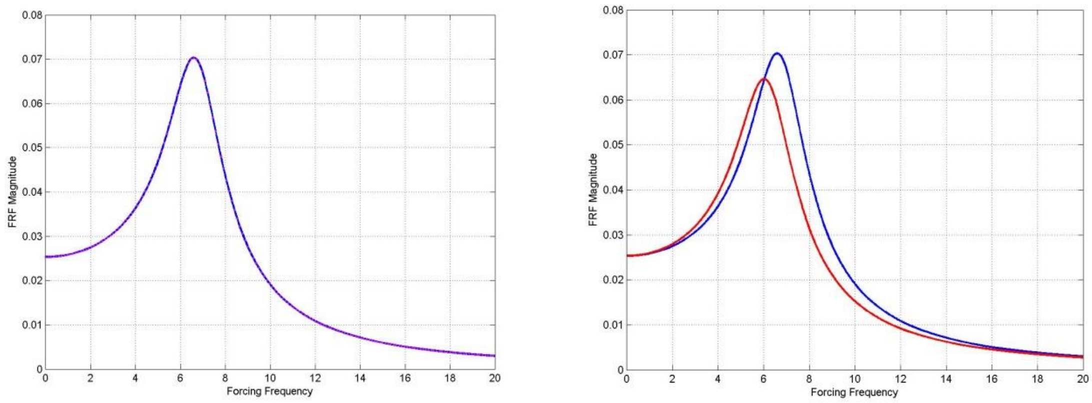

Figure 3. (a) Comparison of FRF with exact techniques. (b) Comparison of FRF with exact and approximate techniques.

2.2 Application of the proposed method to a simple viscoelastic model

Consider the structural model defined in Figure 4: m is de mass associated with the lateral displacement u_1, k_s is the stiffness of the main structure, and the three Kelvin elements in series model a viscoelastic damper (highlighted with dashed green line in the figure).

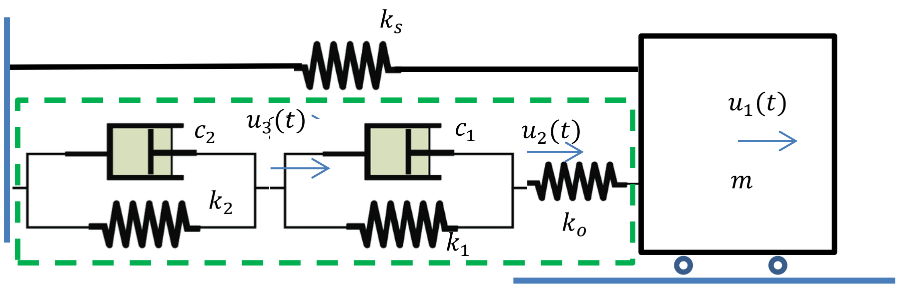

Figure 4. Simple model with viscoelastic damper

The mass, damping and stiffness matrices of the model for the selected displacements ($u_1(t)$, $u_2(t)$, $u_3(t)$) can be assembled for the model to yield:

$$M = \begin{bmatrix} m & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c_1 & -c_1 \\ 0 & -c_1 & c_1 + c_2 \end{bmatrix}, K = \begin{bmatrix} k_s + k_o & -k_o & 0 \\ -k_o & k_o + k_1 & -k_1 \\ 0 & -k_1 & k_1 + k_2 \end{bmatrix} \quad (31)$$

Defining the modal parameters as $m = 1, k_s = 100, k_o = k_1 = k_2 = 50, c_1 = c_2 = 10$ the state-space model matrices computed applying the proposed methodology are:

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -150 & 0 & 50 & 0 \\ 10 & 0 & -15 & 0 \\ 5 & 0 & -5 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \end{bmatrix} \quad (32)$$

The poles of the model are: $s_1 = -0.7528 + 11.3629\mathrm{i}$, $s_2 = -0.7528 - 11.3629\mathrm{i}$,

$s_3 = -13.49$ and $s_4 = -5$ giving a natural frequency of 11.3879 rad/s and modal damping ratio of 0.0661 in the complex poles. As the example illustrates, the proposed analysis technique can be applied to compute the poles in models including any type of viscoelastic model composed by linear viscous dampers and linear elastic springs connected in parallel and series. From the poles, natural frequencies and modal damping ratios can be computed.

3 MODELS WITH LOAD-DEPENDENT RITZ VECTORS

In this section the proposed formulation is combined with order-reduction methods based on Ritz-Lanczos load-dependent vectors to estimate achievable modal damping ratios of a high-rise building with supplemental dampers. The case is included mainly to illustrate the limitation of viscous dampers in modal-damping augmentation due to main structure flexibility of columns and diagonal connectors of the viscous dampers. The structural model used for this application example is the Seascape building, currently under construction in Auckland, New Zealand (Inaudi et al., 2017). The tower has 49 levels above ground and 5 basement levels (with a total height of approximately 190 m above ground, and 20 m x 40 m in plan). Figure 5 shows the first three modes of vibration and natural periods 4.84 s, 4.29 s and 1.74 s for the structure without supplemental dampers (X, Y and R indicate dominant motion in the X-direction, Y-direction or rotation). To improve the performance under wind action of the building, the design objective was to augment the modal damping ratios of the first three modes of vibration of the structure. Both viscous dampers and tuned mass damper alternatives were explored. In this paper a brief description of the achievable modal damping augmentation using viscous dampers is presented.

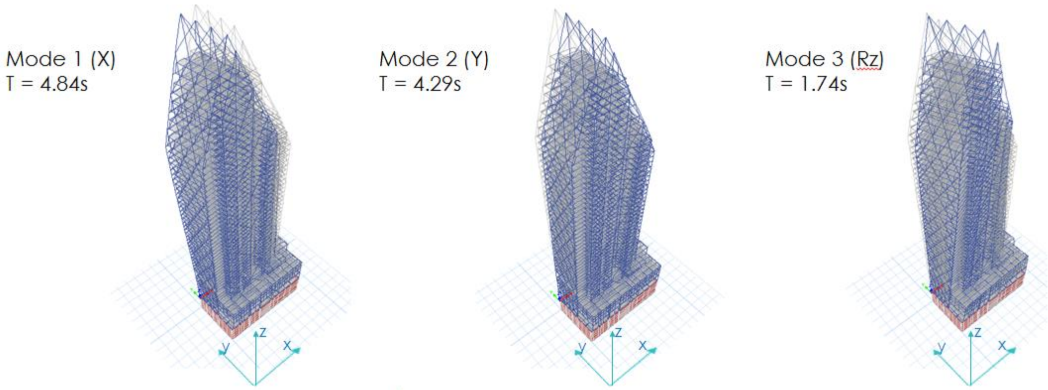

Figure 5. Structural model of Seascape building, modes shapes and natural periods of vibration.

To develop the analysis of the structure with supplemental dampers, a reduced-order model of the frame was obtained using 200 load-dependent vectors (Ritz Lanczos) and computing the mode shapes in the mentioned subspace. This 200 degree-of-freedom model was computed using ETABS® and exported to Matlab® to proceed with pole estimation for different damper configurations. To obtain the load-dependent vectors, the load-influence vectors applied by dampers in diagonal or toggle braces were included. This allows taking into consideration the diagonal and column axial flexibility in the reduced-order model that had not been taken into account if regular modal analysis had been performed with M and K matrices and no load-dependent vectors. The reduced-order model was assumed to have 1% modal damping ratios in all modes of the structure without supplemental dampers (classical modal damping matrix in the generalized coordinates used for the analysis).

Several damper arrangements that best fit with the architectural and structural design of the building were selected for preliminary analysis of maximum reachable modal damping ratios.

Figure 6 shows configuration B (36 dampers) to illustrate a particular damper distribution analyzed: 14 VDs on toggle configuration plus 8 VDs on straight braces on grid line 2 and 14 VDs on toggle configuration on grid line 6 (not shown in the figure). The achievable modal damping in the second mode of vibration for two damper configurations (A and B) are presented.

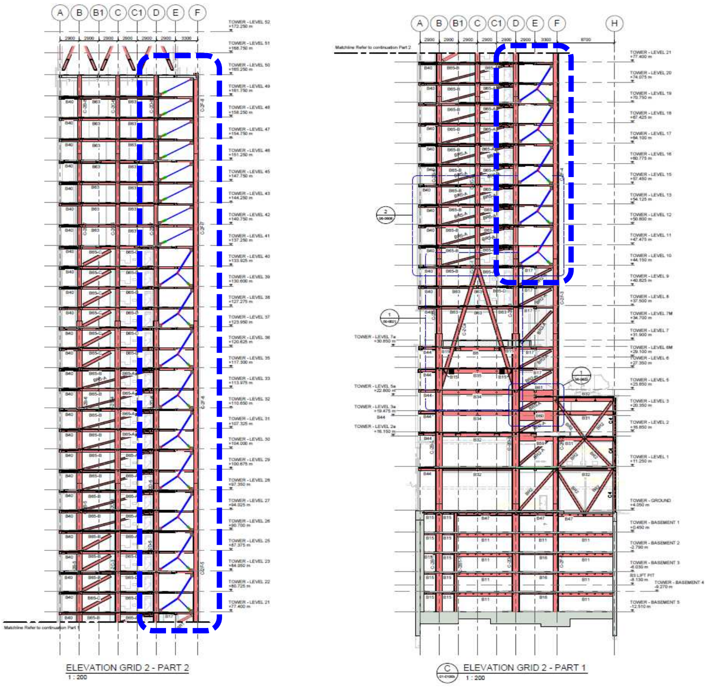

Figure 6. Configuration B of 36 VDs.

Configuration A: For 73 (N_{vd}) selected locations of diagonal and toggle-brace viscous dampers and given structural properties of ETABS model (200 Ritz vector reduced order model) diagonal mass $\boldsymbol{M}_q$ and stiffness matrix $\boldsymbol{K}_q$were extracted from ETABS along with the corresponding 200 mode shapes in the reduced-order subspace of 200 modal coordinates. Assuming $\boldsymbol{C}_s$ as a diagonal damping matrix of the main structure with 1% modal damping ratio and summing the contribution of the supplemental dampers, the total damping matrix of the damped structure was computed as:

$$\boldsymbol{C}_q = \boldsymbol{C}_s + \sum_{j=1}^{N_{vd}} \boldsymbol{L}_{D_jq}{}^T c_j\, \boldsymbol{L}_{D_jq} \tag{33}$$

Where $D_j = \boldsymbol{L}_{D_jq}\boldsymbol{q}$ is the deformation of the j-th damper and $\boldsymbol{L}_{D_jq}$ is the kinematic transformation for the j-th damper. Using a state-space formulation the poles of the non-classically damped model are computed as a function of the damper parameters c_j and the optimization searches for damper parameters that maximize modal damping ratio in the Y-mode (ξ_2). For the optimum damper configuration the maximum reachable ξ_2 and corresponding ξ_1 and ξ_3 values are computed as:

Modal damping ratio in X mode	$\xi_1 = 2.92$ %
Modal damping ratio in Y mode	$\xi_2 = 3.53$ %
Modal damping ratio in torsional mode	$\xi_3 = 4.83$ %

This means that the maximum reachable modal damping ratio ξ_2 for the damper configuration with any viscous parameters is 3.53%. If damper parameters are larger than those of the optimal configuration, this modal damping ratio will decrease.

Configuration B: For 28 VDs on toggle braces plus 8 VDs on diagonal braces (total 36 VDs) selected locations and given structural properties of ETABS model (200 Ritz vector reduced order model) diagonal mass $\boldsymbol{M}_q$ and stiffness matrix $\boldsymbol{K}_q$were extracted from ETABS along with the corresponding 200 mode shapes in the reduced-order subspace of 200 modal coordinates. The damping matrix was assembled using Eq. (33) for $N_{vd} = 36$ and corresponding kinematic matrices. The optimization of ξ_2 in this case gave the following modal damping ratios:

Modal damping ratio in X mode	$\xi_1 = 1.91\%$
Modal damping ratio in Y mode	$\xi_2 = 2.95\%$
Modal damping ratio in torsional mode	$\xi_3 = 4.31\%$

In the case of 36-VDs configuration the optimal modal damping ξ_2 resulted in 2.95% versus 3.53% for the case of 73-VDs configuration. The example shows a case of high-rise building for which a specific modal damping ratio cannot exceed a certain value for a given damper-location configuration due to the effect of axial flexibility of connectors and columns.

4 CONCLUSIONS

An exact model-order reduction technique for linear models with singular mass matrices and viscous or viscoelastic damper has been presented. The technique is particularly useful for the design and response estimation of high-rise buildings with supplemental viscous or viscoelastic dampers. A minimum size state-space formulation is obtained applying the proposed algorithm that allows the computation of exact poles, natural frequencies and modal damping ratios. The use of a large number of Ritz-Lanczos modes including those load pattern vectors associated to the viscous damper forces on the undamped MK model is crucial for the accurate estimation of damping ratios of high-rise buildings with supplemental viscous dampers. The use of a reduced-order model using standard mode shapes of an undamped structure for estimation of modal damping ratios of a structure with supplemental dampers can provide inaccurate modal-damping estimations in high-rise buildings.

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