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MULTIPLE TUNED MASS DAMPERS COUPLED WITH VISCOUS DAMPERS TO IMPROVE ACCELERATION PERFORMANCE AND ROBUSTNESS OF BUILDINGS SUBJECTED TO WIND-INDUCED VIBRATIONS

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Abstract. The performance of a novel configuration of multiple tuned mass damper (called VCMTMD) proposed by the authors for floor acceleration reduction of linear elastic buildings subjected to broad-band wind force excitation is studied. To improve efficiency and robustness under variations or uncertainties of main-structure dynamic parameters, three single degree of freedom mass dampers in parallel are tuned to different frequencies close to the natural frequency of the target mode of vibration of the main structure and the viscous damping elements that connect the tuned mass dampers (TMDs) to the main structure are replaced by dampers that connect the masses of the TMDs. This special coupling between parallel TMDs is called viscous-coupling multiple tuned mass dampers (VCMTMD). The effectiveness of the proposed technique is evaluated and compared with conventional TMD and uncoupled multiple tuned mass damper (MTMD) configurations for the same total mass of the TMDs. The reduction of root mean square acceleration of the main structural system subjected to broad-band excitation is analyzed and compared with those of a conventional optimal TMD and a parallel MTMD without viscous coupling. The VCMTMD outperforms both the conventional TMD and parallel MTMDs in terms of acceleration reduction of the structure with the proposed VCMTMD configuration. The VCMTMD shows high robustness to changes or uncertainties in main-structure parameters (mass or stiffness). The proposed TMD configuration can find practical applications in wind-induced acceleration reduction in high-rise buildings and vibration reduction in other types of structures subjected to broad-band loading.

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1 INTRODUCTION

The use of tuned mass dampers (TMD) to increase damping ratios of specific modes of vibration of an undamped main structure or a structure with low modal damping ratios subjected to broad-band excitation has been proposed and applied during the last decades around the world in different structures and mechanical systems (Warburton, 1982; Fischer 2007; Nakai et al, 2019; Inaudi et al., 2017).

The most frequent technology proposed to provide robustness of performance under changes in main structure parameters is the multiple tuned mass damper (MTMD) configuration: several TMDs connected in parallel with different frequencies tuned in an interval close to the natural frequency of the target mode of the structure. Igusa and Xu, 1994 and Bakre and Jangid, 2007 have shown that MTMD can be more effective and robust than a single TMD with the same total mass. Different configurations have been proposed and studied in MTMD (Asami, 2017; Kareem and Kline, 1995).

A novel configuration of MTMD (Figure 1c) is considered in this paper to improve performance and lower sensibility to changes in main structure parameters. The idea is to use a set of closely-spaced natural-frequency undamped TMDs connected in parallel to the main structure with viscous dampers that couple the TMDs between them.

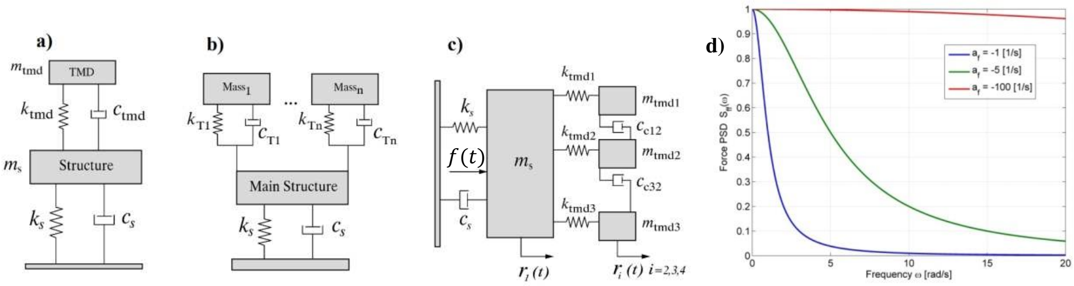

Figure 1. a) Main structure with conventional TMD, b) with MTMD and c) with proposed VCMTMD (Inaudi, 2022). d) External force power spectral density model used in this study.

In section 2 we analyze the optimum performance of conventional TMD connected to a main structure subjected to broad-band external force with power spectral density (PSD) defined in Figure 1c. The optimum tuning parameters of the TMD are compared with analytical expressions of the optimum parameters for main-structure deformation reduction given by Warburton (1982) for white-noise input. The optimum tuning parameters and performance of a MTMD configuration with three TMDs operating in parallel and closely spaced natural frequencies is explored and compared to conventional TMD in Section 3. Next, the proposed viscously coupled multiple TMDs performance is analyzed in Section 4. Finally, the robustness of the three configurations is compared in Section 5.

2 TMD OPTIMUM PARAMETERS FOR ACCELERATION REDUCTION

In cases of occupant comfort protection of high-rise buildings subjected to wind-induced reduction applications, the wind loads are broad-band processes that shows decreasing PSD with increasing frequency. The optimum design criterion for occupant comfort is not rms deformation, but rms acceleration, mean peak acceleration, or one-third octave rms filtered acceleration spectrum (performance variables usually used to assess occupant comfort). In this case, optimal tuning parameters can be searched by computational optimization for the corresponding forcing model and performance index, using Warburton's parameters as starting values in the iterative procedure.

Let us consider a simple model of TMD attached to a single-degree of freedom model of a main structure subjected to an external load, $f(t)$, applied to the main structure. The equations of motion of the model are

$$M\,\ddot{r}(t) + C\,\dot{r}(t) + K\,r(t) = L_f f(t) \tag{1}$$

$$M = \begin{bmatrix} m_s & 0 \\ 0 & m_{tmd} \end{bmatrix},\; C = \begin{bmatrix} c_s + c_{tmd} & -c_{tmd} \\ -c_{tmd} & c_{tmd} \end{bmatrix},\; K = \begin{bmatrix} k_s + k_{tmd} & -k_{tmd} \\ -k_{tmd} & k_{tmd} \end{bmatrix},\; L_f = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \tag{2}$$

Where m_s, c_s, k_s are the mass, viscous constant, and stiffness of the main-structure model. The frequency response function (FRF) of the coupled main structure-TMD model from applied load to main structure deformation can be obtained. The absolute acceleration of the main structure subjected to white noise external force $f(t)$ it is not a second-order process (finite variance) due to the contribution of the white noise signal $f(t)$ (infinite variance random process) to the absolute acceleration of the structure, $\ddot{r}_1(t)$:

$$\ddot{r}_1(t) = [\,1 \quad 0]\,M^{-1}\,(-K\,r(t) - C\,\dot{r}(t) + L_f\,f(t)) \tag{3}$$

To analyze optimum TMD for acceleration reduction of the main structure, we need a finite variance model for the broad-band external load. With this objective, and considering that in wind applications, the power spectral density of the wind load $f(t)$ decreases with frequency, a simple filtered white-noise process is proposed to analyze the acceleration response of a structure:

$$\dot{x}_f(t) = a_f\,x_f(t) + b_f\,w(t) \qquad f(t) = x_f(t) \tag{4}$$

$$R_{ww}(\tau) = W\,\delta(\tau) \tag{5}$$

The PSD of the stationary force process, $S_{ff}(\varpi)$, can be computed as

$$S_{ff}(\varpi) = \frac{{b_f}^2}{\varpi^2 + {a_f}^2}\,\frac{1}{2\pi}\,W \tag{6}$$

where W is the white-noise intensity. Figure 1d shows the force PSD $S_{ff}(\varpi)$ for $a_f = -1, -5, -100$, $b_f = -a_f/\sqrt{2\pi}$ and $W = 1$. As $|a_f|$ tends to zero, the power spectral density of the force process approaches $S_{ww}(\varpi)$ of white noise. For small values of $|a_f|$ the process can be tailored to exhibit a decreasing PSD with frequency in the range of interest (for wind applications between 0 and 20 rad/s). This study uses this simple filter to allow the use of the Lyapunov equation to compute mean square acceleration and explore optimum parameter changes in the design of conventional TMDs and other configurations. The force rms, σ_f, of each of these models depends on the filter parameters and W, reducing significantly as $|a_f|$ tends to zero and increasing with $\sqrt{W}$. As the optimum TMD parameters in linear models do not depend on σ_f but on the frequency content of the force random process, the proposed model allows the consideration of the influence of the frequency content on those parameters. More sophisticated PSD and other filtered white noise models could be developed to model wind forces applied to structures, but the proposed filter allows us an initial study of the effect of decreasing PSD with frequency on the optimal parameters of TMDs.

The extended state-space formulation of the main structure with TMD subjected to the filtered white noise force can be expressed as

$$\dot{x}_e(t) = A_e\,x_e(t) + B_e\,w(t) \tag{7}$$

where

$$x_e(t) = \begin{bmatrix} r(t) \\ \dot{r}(t) \\ x_f(t) \end{bmatrix},\quad A_e = \begin{bmatrix} O & I & O \\ -M^{-1}K & -M^{-1}C & M^{-1}L_f \\ 0 & 0 & a_f \end{bmatrix},\; B_e = \begin{bmatrix} 0 \\ 0 \\ b_f \end{bmatrix} \tag{8}$$

Where $w(t)$ is a stationary white-noise process with an autocorrelation function defined in Eq (5). The outputs of interest, deformation rms, and acceleration rms of the main structure can be expressed as

$$\sigma_{r_1} = \sqrt{D_r\, P_{x_e x_e} D_r^{\,T}} \qquad \sigma_{\ddot{r}_1} = \sqrt{D_{\ddot{r}_1}\, P_{x_e x_e} D_{\ddot{r}_1}^{\,T}} \tag{9}$$

Where $P_{x_e x_e}$ is the stationary covariance matrix of the extended state x_e and

$$D_r = [1 \quad 0 \quad 0 \quad 0 \quad 0] \qquad D_{\ddot{r}_1} = [1 \quad 0][-M^{-1}K \quad -M^{-1}C \quad M^{-1}L_f] \tag{10}$$

$P_{x_e x_e}$ can be computed using the algebraic Lyapunov equation

$$A_e(\mu, \beta,\, \xi_{tmd}) P_{x_e x_e} + P_{x_e x_e} A_e(\mu, \beta,\, \xi_{tmd})^T + B_e\, W\, B_e^{\,T} = 0 \tag{11}$$

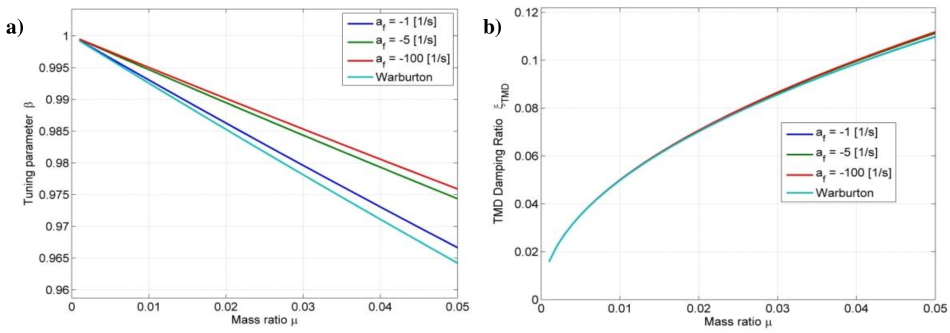

Figure 2. Optimum $\boldsymbol{\beta}$ and ξ_{tmd} parameters for conventional TMD. Comparison with Warburton's parameters

In Figures 2a and 2b we compare the optimum parameters of the TMD for filtered white noise input computed by numerical optimization with rms-structure-acceleration performance index and Warburton´s optimal parameters defined for rms-structure-deformation performance. The figure shows the optimum parameters for the three input models for an undamped main structure, with $\boldsymbol{\omega_s = 2\ \mathrm{rad/s}}$. The optimum TMD tuning parameter $\boldsymbol{\beta}$ *f*or acceleration rms reduction for filtered-white noise is slightly larger than Warburton's value (Figure 2a). The larger the absolute value of $|\mathbf{a_f}|$, the larger the difference between these parameters. Optimum TMD damping ratios for filtered white-noise input are very similar to Warburton's value for white-noise input for the range of mass ratios analyzed (Figure 2b). In any case, Warburton's optimum parameters are good estimators of the optimum tuning parameters for filtered white-noise force and rms acceleration performance index.

3 MULTIPLE TUNED MASS DAMPERS IN PARALLEL (MTMD)

MTMDs have been proposed to increase the frequency bandwidth of the imaginary part of the dynamic stiffness of the MTMD around a main structural frequency to improve robustness under uncertainty or changes in the dynamic properties of the main structure around nominal values. In this case, tuning of the TMDs is achieved with closely spaced natural frequencies of each TMD. This application is the focus of this section. To analyze the feasibility of improving the performance and robustness of the conventional single TMD, we analyze a case of triple TMD (Figure 1b) with identical masses $m_{tmd1} = m_{tmd2} = m_{tmd3} = m_{tmd}/3$ equal to the third of the mass m_{tmd} of a conventional TMD. Each TMD, labeled as 1, 2, and 3, is defined with different natural frequencies: $\omega_{tmd2} =$ central frequency equal to the optimal frequency of the conventional TMD ω_{tmd} multiplied by a shifting parameter α_ω for a given mass ratio, $\omega_{tmd1} = (1 + \Delta_\omega)\, \omega_{tmd2}$, $\omega_{tmd3} = (1 - \Delta_\omega)\omega_{tmd2}$ where $\Delta_\omega$is a dimensionless parameter that controls the bandwidth of the MTMD. For simplicity, all TMDs are assigned with the same damping ratios $\xi_{tmd1} = \xi_{tmd2} = \xi_{tmd3} = \xi_{tmd}$. Even though the mass, damping, and stiffness properties of each mass damper of the MTMD could be defined as independent variables for the optimization, the selected parameter configuration allows a simpler optimization process with three parameters: Δ_ω, ξ_{tmd} and α_ω. $\omega_{tmd} = \alpha_\omega\, \beta\, \omega_{st}$ is the optimal frequency of TMD2, β is the optimum tuning parameter defined by Warburton in (Table 1), and α_ω is a shifting frequency parameter close to one that improves tuning for the parallel TMDs. The equations of motion of the model subjected to broad-band excitation applied on the main structural system can be obtained as follows:

$$M_{mtmd}\, \ddot{r}(t) + C_{mtmd}\, \dot{r}(t) + K_{mtmd}\, r(t) = L_w\, w(t) \tag{12}$$

where

$$M_{mtmd} = \begin{bmatrix} m_{st} & 0 & 0 & 0 \\ 0 & m_{tmd}/3 & 0 & 0 \\ 0 & 0 & m_{tmd}/3 & 0 \\ 0 & 0 & 0 & m_{tmd}/3 \end{bmatrix} \tag{13}$$

$$C_{mtmd} = \begin{bmatrix} c_{st} + c_{tmd1} + c_{tmd2} + c_{tmd3} & -c_{tmd1} & -c_{tmd2} & -c_{tmd3} \\ -c_{tmd1} & c_{tmd1} & 0 & 0 \\ -c_{tmd2} & 0 & c_{tmd2} & 0 \\ -c_{tmd3} & 0 & 0 & c_{tmd3} \end{bmatrix} \tag{14}$$

$$K_{mtmd} = \begin{bmatrix} k_{st} + k_{tmd1} + k_{tmd2} + k_{tmd3} & -k_{tmd1} & -k_{tmd2} & -k_{tmd3} \\ -k_{tmd1} & k_{tmd1} & 0 & 0 \\ -k_{tmd2} & 0 & k_{tmd2} & 0 \\ -k_{tmd3} & 0 & 0 & k_{tmd3} \end{bmatrix},\ L_w = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

Where the total TMD mass is defined as $3m_{tmd} = \mu\, m_{st}$. According to the defined dimensionless parameters in this case

$$k_{tmd1} = \omega_{tmd}{}^2\, (1 + \Delta_\omega)^2\, m_{tmd}/3 \qquad k_{tmd2} = \omega_{tmd}{}^2\, m_{tmd}/3 \qquad k_{tmd3} = \omega_{tmd}{}^2\, (1 - \Delta_\omega)^2\, m_{tmd}/3 \tag{16}$$

$$c_{tmd1} = \frac{2\xi_{tmd}\omega_{tmd}\, (1+\Delta_\omega)\, m_{tmd}}{3} \qquad c_{tmd2} = \frac{2\xi_{tmd}\, \omega_{tmd}\, m_{tmd}}{3} \qquad c_{tmd3} = \frac{2\xi_{tmd}\, \omega_{tmd}\, (1-\Delta_\omega)\, m_{tmd}}{3} \tag{17}$$

The damping ratios of all TMDs ξ_{tmd} are assumed to be equal for the three TMDs as a function of the total mass ratio. Table 1 presents the analytical expressions derived by Warburton to estimate the optimal fitting parameter of frequency and damping ratios. Assuming a filtered white-noise input, the optimal shifting-frequency parameter α_ω, bandwidth parameter Δ_ω, and damping ratios ξ_{tmd} of the parallel TMDs can be computed numerically, minimizing the structure-acceleration rms (defined in Eq. 9). This can be done by solving the stationary Lyapunov equation as in the case of the structure-TMD model analyzed in the previous section, defining the appropriate state vector, assembling the corresponding system matrices A and B using the mass, damping, and stiffness matrices defined in Eqs. (13,14,15).

Criterion	$\beta = \dfrac{\omega_{tmd}}{\omega_{st}}$	ξ_{tmd}
RMS deformation reduction of undamped main structure subjected to white noise force as a function of mass ratio	$\dfrac{\sqrt{1-\mu/2}}{1+\mu}$	$\sqrt{\dfrac{\mu(4+3\mu)}{8(1+\mu)(2+\mu)}}$

Table 1. Analytical formulations for tuning parameters under an external load applied to the main structure.

Figures 3a, 3b and 3c show the optimal frequency-shift parameter α_ω, the optimum bandwidth parameter Δ_ω, and the optimum damping ratio ξ_{TMD} as functions of the total mass ratio μ of the MTMD, for the undamped main structure ($\xi_s = 0$) for different broad-band models of the external force, considering rms acceleration of the main structure as the performance index. The optimal frequency-shift parameter takes values close to one, increasing linearly with the total mass ratio μ (see Figure 3a). The optimal bandwidth parameter for rms structure-acceleration reduction increases with mass ratio and is not sensible to filter parameter a_f as shown in Figures 3b.

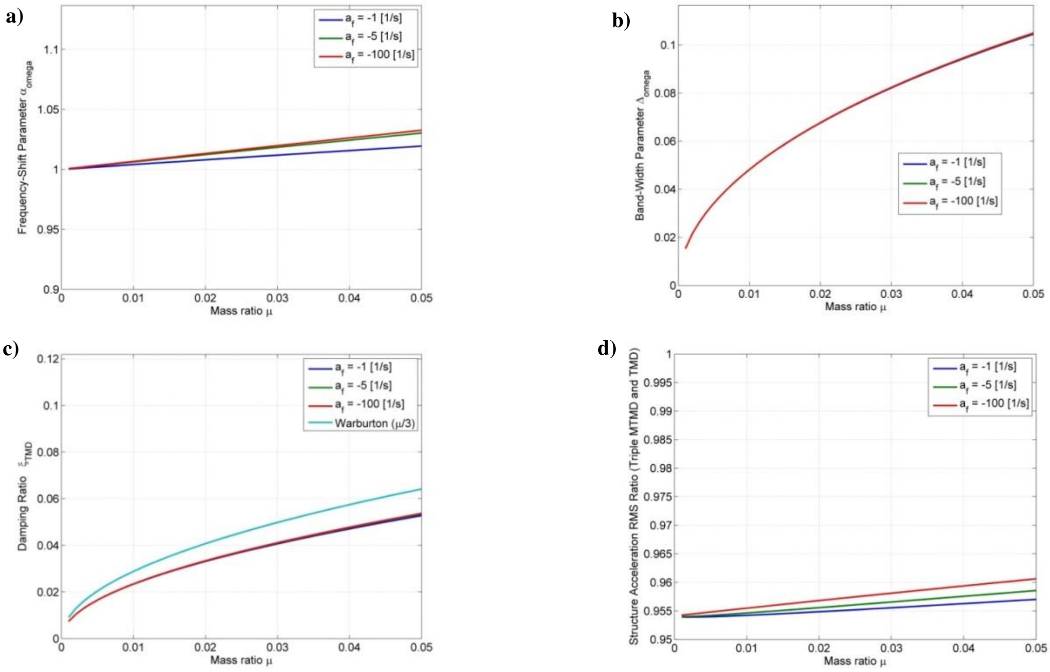

Figure 3. Optimum parameters and performance comparison of MTMD for different filtered-white noise models.

Figure 2c illustrates that the optimum damping ratios for MTMD are slightly smaller than Warburton optimum damping ratio computed for $\mu/3$. Recall that μ is defined as the ratio of the total mass of the 3 TMD operating in parallel and the main structure mass. These optimum damping ratios are not sensible to forcing frequency content for the range of filter parameters considered. Figure 2d shows the rms acceleration of the structure with optimum MTMD normalized by the corresponding rms acceleration of the structure with optimum TMD. As the figure indicates, performance improvement (ratios smaller than one) can be achieved with MTMDs with respect to a conventional TMD for the range of mass ratio considered typical in wind-induced vibration reduction applications. Depending on the value of a_f and the total mass ratio μ of the 3 TMDs, the improvement is in the range of 4.5% to 4.8% for $\mu < 0.01$, and reduces to 3.9% to 4.4% for $\mu = 0.05$. For the cases of larger mass ratios, the constraint imposed of equal damping ratios of individual TMDs of the MTMD should be released to have these parameters as free variables of the optimization process. This would enable improved acceleration performance of the MTMD. We do not include the results of such analysis to focus on the proposed novel configuration of VCMTMD.

4 MTMDS COUPLED WITH VISCOUS DAMPERS (VCMTMD)

The proposed MTMD consists of a variation of a MTMD in parallel with viscous coupling among the TMDs and no viscous damping in the TMD with central frequency. This configuration was proposed by Inaudi 2022. As illustrated in Figure 1c, no elastic elements are used connecting the 3 TMDs to avoid the appearance of spurious natural frequencies. All TMDs are designed without a damper connection to the structure in the seminal conceptual configuration. The effect on performance of the direct-damper connection along with inter-TMD viscous coupling will be explored in future research to assess whether further improvement could be achieved.

The equations of motion of the VCMTMD-main structure system are

$$M_{mtmd}\ddot{r}(t) + C_{vcmtmd}\dot{r}(t) + K_{mtmd}r(t) = L_w\, f(t) \tag{18}$$

where

$$M_{mtmd} = \begin{bmatrix} m_{st} & 0 & 0 & 0 \\ 0 & m_{tmd}/3 & 0 & 0 \\ 0 & 0 & m_{tmd}/3 & 0 \\ 0 & 0 & 0 & m_{tmd}/3 \end{bmatrix} \tag{19}$$

$$C_{vcmtmd} = \begin{bmatrix} c_{st} + c_{tmd1} + c_{tmd3} & -c_{tmd1} & -c_{tmd2} & -c_{tmd3} \\ -c_{tmd1} & c_{tmd1} & 0 & 0 \\ -c_{tmd2} & 0 & c_{tmd2} & 0 \\ -c_{tmd3} & 0 & 0 & c_{tmd3} \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & c_{c12} & -c_{c12} & 0 \\ 0 & -c_{c12} & c_{c12} + c_{c23} & -c_{c23} \\ 0 & 0 & -c_{c23} & c_{c23} \end{bmatrix} \tag{20}$$

$$K_{mtmd} = \begin{bmatrix} k_{st} + k_{tmd1} + k_{tmd2} + k_{tmd3} & -k_{tmd1} & -k_{tmd2} & -k_{tmd3} \\ -k_{tmd1} & k_{tmd1} & 0 & 0 \\ -k_{tmd2} & 0 & k_{tmd2} & 0 \\ -k_{tmd3} & 0 & 0 & -k_{tmd3} \end{bmatrix}, L_w = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{21}$$

The strategy looks for pole damping ratio augmentation of poles with frequencies close to the central frequency of the VCMTMD. The improved main structure performance is expected resulting from an energy dissipation mechanism due to interaction between TMDs, not only due to relative deformation of the TMDs with respect to the support point. For this reason, the direct damping constants of the 3 TMDs are assumed to be zero ($c_{tmd1} = c_{tmd2} = c_{tmd3} = 0$) as shown in Figure 1c. The seminal idea (Inaudi, 2022) was to change the energy dissipation mechanism of the MTMD: using relative displacements between parallel TMDs with coupling dampers instead of dissipation with direct dampers connecting the structure to each TMD. Incorporating coupling dampers and lowering or eliminating direct viscous dampers of each TMD allows the dynamic stiffness of the MTMD increases significantly its imaginary component for a frequency band centered at the central frequency of the second TMD, tuned close to the main structure natural frequency. Although more parameters of the VCMTMD could be optimized (for example, individual variation of each damping ratio of the TMDs 1 and 3, a shift in the central frequency of TMD 2, and different damping ratios for TMDs 1 and 3) for the sake of simplicity the coupling damping terms are defined equal ($c_{c23} = c_{c12} = c_c$), direct TMD damping ratios are defined as $\xi_{tmd1} = \xi_{tmd3} = \xi_{tmd3} = \Delta_\xi \xi_{tmd}$, with Δ_ξ defined as a direct TMD damping parameter, and ξ_{tmd} the optimal conventional TMD damping ratio for the total mass considered defined by Warburton's formula (see Table 1).

The central frequency is defined using Warburton's optimal tuning factor $\beta = \omega_{tmd2}/\omega_s$ computed for conventional TMD for the total mass ratio μ multiplied by a frequency-shift parameter α_ω as defined in the case of MTMD. For given values of mass ratio μ and Δ_ξ, the optimization is done by searching for optimal frequency-shift parameter α_ω, optimal frequency bandwidth parameter Δ_ω (defined as in the case of MTMDs), and an additional dimensionless parameter Δ_c that defines the viscous coupling term c_c between TMDs 1 and 2 and between TMDs 2 and 3 as a function of the optimal damping ratio of the classical TMD model defined by Warburton and evaluated with the formula shown below:

$$\Delta_c = \frac{c_c}{2\, m_{tmd}(\mu)\, \xi_{tmd}(\mu)\, \beta(\mu)\, \alpha_\omega \omega} \tag{22}$$

Figures 4a, 4b and 4c show the computed optimal frequency-shift parameter α_ω, the optimal bandwidth parameter Δ_ω and the optimal parameter Δ_c of the VCMTMD as functions of the mass ratio for the design with $\Delta_\xi = 0$ (no direct TMD dampers connecting TMDs and main structure). Both α_ω and Δ_ω increase monotonically with μ for all filters considered. The frequency-shift parameter takes values slightly larger than one. We can observe that Δ_ω is significantly larger than that of MTMD for all mass ratio range considered. The parameter Δ_c for rms acceleration reduction shows that it is sensitive to the filter parameters and that its optimal values vary linearly with the mass ratio in the range $0 < \mu < 0.05$ considered.

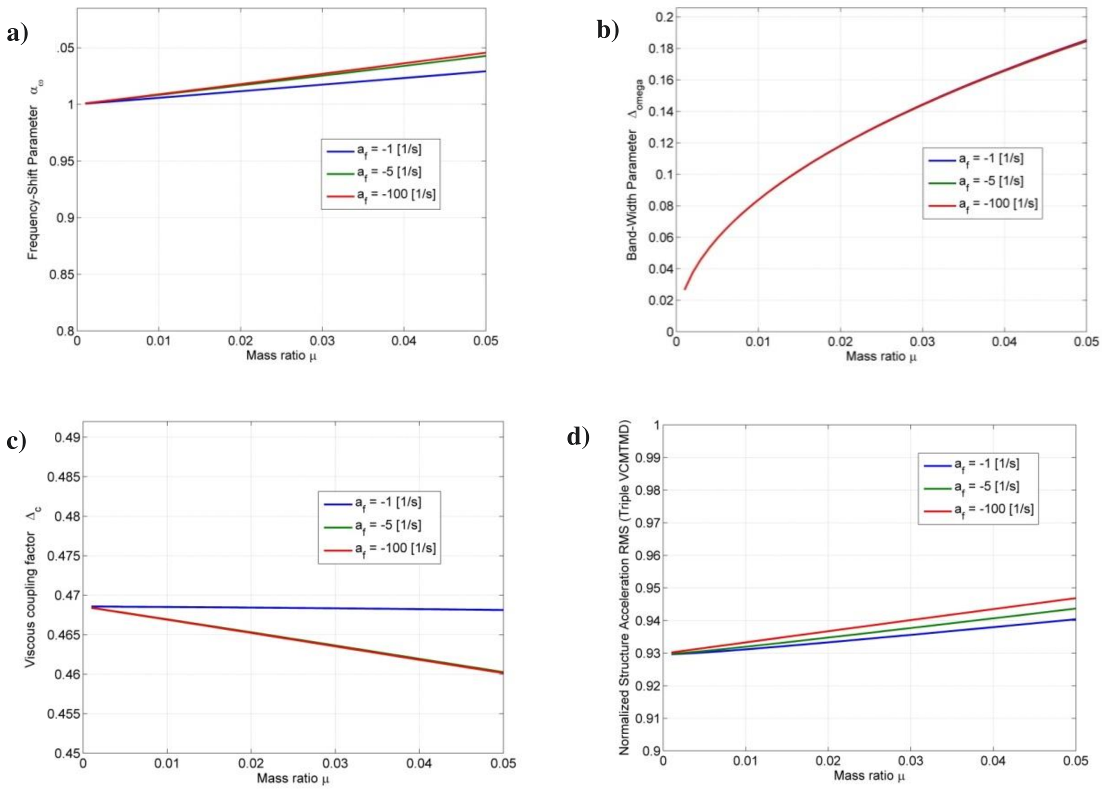

Figure 4. Optimal parameters for VCMTMD in the undamped main structure and direct damping $\Delta\xi$=0 and normalized optimal rms acceleration of the main structure for VCMTMD.

Figure 4d compares the achievable optimal performance of the structure with optimum VCMTMD normalized by the main structure rms acceleration of the model with optimal TMD for the same total mass in the devices (same μ). Comparing Figure 3d and Figure 4d we notice that VCMTD outperforms MTMD for all mass ratios. Performance improvement can be achieved with VCMTMDs with respect to both conventional TMD and MTMDs for the range of mass ratio considered typical in wind-vibration reduction applications. Depending on the value of a_f and the total mass ratio μ of the 3 VCTMDs, the improvement is in the range of 6.7% to 7% for $\mu < 0.01$, and reduces to 5.4% to 6% for $\mu = 0.05$. For the cases of larger mass ratios, the imposed constraint of equal damping ratios of individual VCTMDs should be released to have these parameters as free variables of the optimization process. This would enable improved acceleration performance of the VCMTMD. Because mass ratios of individual TMDs in the case of MTMD or VCTMD are a third of that of a conventional TMD, rms deformations of each TMD are larger than those of a comparable TMD with the same total mass. This implies a more significant space requirement for operation of MTMD and VCTMD compared to conventional single TMD. Although not shown in figures, optimum VCMTMD rms TMD-deformations are smaller than those of parallel uncoupled MTMD for the same total mass. This implies that optimum VCMTMD not only provides a better rms acceleration of the main structure but also has a smaller space requirement for operation if individual TMDs are configured side by side.

5 SENSITIVITY OF TMD DESIGN FOR NOMINAL DESIGN

To analyze the robustness of the three configurations of TMDs to main structure changes in stiffness or mass parameters, we analyze the sensitivity of rms acceleration performance for optimum conventional TMD, MTMD, and VCMTD with the same total mass ratio. The rms acceleration of the structure-TMD model is computed for small changes in structural stiffness and mass around nominal values (${}_{nom}k_s$, ${}_{nom}m_s$) with an optimal TMD, MTMD, or VCMTD for the nominal structural model. Stiffness and mass variations of the main structural system are defined with dimensionless parameters α_{k_s} and α_{m_s}, such that

$$k_s = {}_{nom}k_s \, \alpha_{k_s} \qquad m_s = {}_{nom}m_s \, \alpha_{m_s} \tag{23}$$

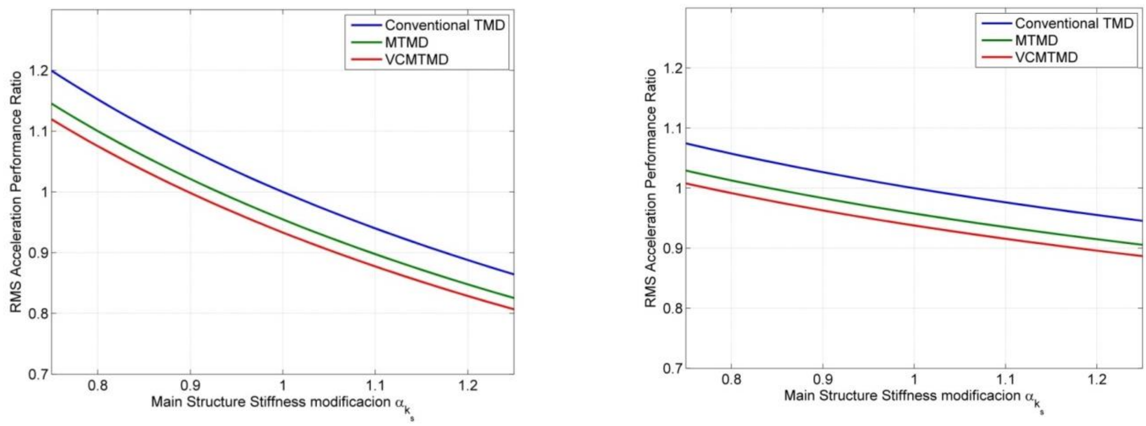

Figure 5. Normalized RMS Acceleration Sensitivity of the Main Structure for TMD, MTMD, and VCMTMD with Normalized rms acceleration of main structure sensitivity for TMD, MTMD and VCMTMD with Δξ=0 under filtered white noise loading as a function of the stiffness variation parameter. a) $\mathbf{a_f = -100\ 1/s}$ b) $\mathbf{a_f = -1\ /s}$.

Figures 5a and 5b show the performance sensitivity to changes of the nominal value of the stiffness of the main structural system subjected to filtered white noise with parameters $a_f = -100, -1$ and $b_f = -a_f$ for the optimal TMD, MTMD and VCTMD designed for the nominal parameters. In both figures, the rms acceleration of the main structure of all cases is normalized with respect to the rms acceleration of the main structure with the optimal TMD designed with the nominal parameters of the main structure. The results show that the performance sensitivity for all TMD, MTMD, and VCMTMD are comparable, but in all cases, VCMTMD outperforms the conventional TMD and the parallel uncoupled MTMDs. The same condition is observed for changes of the nominal value of the mass of the main structural system.

6 CONCLUSIONS AND FUTURE RESEARCH

A novel configuration of multiple tuned mass dampers coupled by viscous dampers between TMDs has been proposed for acceleration reduction of linear elastic structures subjected to broad-band excitation. Three single degree of freedom mass dampers in parallel are tuned to frequencies close to the natural frequency of the target mode of vibration of the main structure, and the viscous damping elements connect the masses of the TMDs: central frequency TMD2 is connected with higher frequency TMD1 and with lower frequency TMD3. The optimum TMD central frequency parameter, frequency bandwidth of TMDs, and optimum damping ratio are computed using the rms main structure acceleration as the performance index for the structure subjected to broad-band excitation. The excitation is modeled as filtered white noise in a state-space model formulation to facilitate the estimation of stationary response using the algebraic Lyapunov equation in the computation of the state covariance matrix of the extended model.

The effectiveness of the proposed MTMD is evaluated and compared with conventional TMD and uncoupled MTMD configurations for the same total mass of the TMDs. The proposed VCMTMD is more efficient than a conventional TMD or uncoupled MTMD for the same total mass of the TMD device and broad-band excitation. The VCMTMD outperforms both the conventional TMD and parallel MTMDs in terms of acceleration reduction and shows high robustness to changes/uncertainties in main structure parameters (mass or stiffness).

The results of this research indicate that the proposed configuration is a promising energy dissipation strategy for structures operating in the linear range subjected to broad-band excitation. The construction of mechanical prototypes that implement the proposed concept of coupled TMDs and an experimental program to verify performance and its technological feasibility are lines of future research to be pursued by the authors. A broader optimization of the parameters of the proposed VCMTMD that allows the selection of different masses in the TMDs, different values of direct and coupling viscous damping parameters in TMDs, as well as not equally spaced frequencies of the TMDs could improve even more the acceleration performance of the main structure. Another line of numerical and experimental research to be explored is the potential benefits of the combination of VCMTMD with inerters to reduce the total mass of the energy-dissipation device maintaining acceleration-reduction capability.

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